Applied Game Theory in Business Analytics

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INTRODUCTION

Conflict has been a central theme in human history. Conflict arises when two or more individuals with different views, goals, or aspirations compete to control the course of future events. Game theory studies competition. It uses mathematics and mathematical tools to study situations in which rational players are involved in conflict both with and without cooperation. According to Weins (2003), game theory studies situations in which parties compete, and also possibly cooperate, to influence the outcome of the parties' interaction to each party's advantage. The situation involves conflict between the participants called players because some outcomes favor one player at the possible expense of the other players. What each player obtains from a particular outcome is called the player's pay-off. Each player can choose among a number of strategies to influence his pay-off. However, each player's pay-off depends on the other players' choices. According to Straffin (2004) rational players desire to maximize their own payoffs. Game theory is a branch of applied mathematics that is used in the social sciences (most notably in economics), business, biology, decision sciences, engineering, political science, international relations, operations research, applied mathematics, computer science, and philosophy. Game theory mathematically captures behavior in strategic situations in which an individual's success in making choices depends on the choices of others. Although initially developed to analyze competitions in which one individual does better at another's expense, game theory has grown to treat a wide class of interactions among players in competition.

Explanation of Game Theory Features

Games can have several features; a few of the most common are listed here.

- **Number of Players:** Each person who makes a choice in a game or who receives a payoff from the outcome of those choices is a player. A two-person game has two players. A three or more person is referred to as an N-person game.
- **Strategies per Player:** Each player chooses from a set of possible actions, known as strategies. In a two person game we allow the row player to have up to m strategies and the column player to have up to n strategies. The choice of a particular strategy by each player determines the pay-off to each player.
- **Pure Strategy Solution:** If a player should always choose one strategy over all other strategies to obtain their best outcome in a game, then that strategy represents a pure strategy solution. Otherwise if strategies should be played randomly then the solution is a mixed strategy solution.
- **Nash Equilibrium:** A Nash equilibrium is a set of strategies which represents mutual *best responses* to the other player's strategies. In other words, if every player is playing their part of Nash equilibrium, no player has an incentive to unilaterally change his or her strategy. Considering only situations where players play a single strategy without randomizing (a pure strategy) a game can have any number of Nash equilibrium.

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- **Sequential Game:** A game is sequential if one player performs her/his actions after another; otherwise the game is a simultaneous game.
- **Simultaneous Game:** A game is simultaneous if the players each choose their strategy for the game and implement them at the same time.
- **Perfect Information:** A game has perfect information if either in a sequential game every player knows the strategies chosen by the players who preceded them or in a simultaneous game each player knows the other players strategies and outcomes in advance.
- **Constant Sum or Zero-Sum:** A game is constant sum if the sums of the payoffs are the same for every set of strategies and zeros-um if the sum is always equal to zero. In these games one player gains if and only if another player loses otherwise we have variable sum game.
- **Extensive Form:** Presents the game in a tree diagram while normative form presents the game in a payoff matrix. In this chapter we only present the normative form and its associated solution methodologies.
- **Outcomes:** an outcome is a *set* of *strategies* taken by the players, or it is their payoffs resulting from the actions or strategies taken by all players.
- **Total Conflict Games:** Are games between players where the sums of the outcomes for all strategy pairs are either the same constant or zero. Games whose outcome sums are variable are known as partial conflict games.

The study of game theory has provided many classical and standard games that provide insights into the games. Table 1 provides a short summary of some of these games.

This chapter is primarily concerned with two-person games. The irreconcilable, conflicting interests between the two players in a game resemble parlor games and military encounters between enemy states. Giordano et al. (2013) explain the two person game in a context of modeling. Players make moves and counter-moves, until the rules of engagement declare the game is ended. The rules of engagement determine what each player can or must do at each stage (the available and/or required moves given the circumstances of the game at this stage) as the game unfolds. For example, in the game rock, paper, scissors both

Table 1. Summary of classical games in game theory

Game	Players	Strategies per Player	Number of Pure Strategy Nash Equilibrium	Sequential	Perfect Information	Zero Sum
Battle of the Sexes	2	2	2	No	No	No
Blotto Games	2	variable	variable	No	No	Yes
Chicken (aka Hawk-Dove)	2	2	2	No	No	No
Matching Pennies	2	2	0	No	No	Yes
Nash Bargaining Game	2	infinite	infinite	No	No	No
Prisoner's Dilemma	2	2	1	No	No	No
Rock, Paper, Scissors	2	3	0	No	No	Yes
Stag Hunt	2	2	2	No	No	No
Trust Game	2	infinite	1	Yes	Yes	No

A complete list maybe viewed at http://en.wikipedia.org/wiki/List_of_games_in_game_theory.