Equilibria for Traffic Network with Capacity Constraints

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INTRODUCTION

There have been several decades in the study of equilibria for optimizations related to road traffic due to its importance in practical applications and theoretical challenges. Wardrop (1952) studied the theoretical properties from mathematical and statistical viewpoints; since then many publications have been developed in this area. The classic book by Beckmann, McGuire and Winsten (1956) provides us equilibrium of traffic optimization with travel cost function. Quandt (1967) and Schneider (1968) first studied the equilibrium problem for traffic network with multicriteria. Dafermos (1972) studied traffic assignment problem in a network of multiclass-user model with travel units were divided into different classes, and an algorithm was given in this paper for optimizing the traffic patterns. Toll policies of the traffic network for multiclass-user were also studied (Dafermos, 1973). The conditions for the existence, uniqueness and stability of equilibria were studied by Smith (1979). Fisk introduced a network optimization problem for the traffic assignment with congestion effects, and the solution was then shown to be an equilibrium (Fisk, 1980). Wardrop's equilibrium principle has been generalized to vector space (Chen & Yen, 1993), but the equivalence between the traffic equilibrium and variational inequality does not hold any more. Therefore, it is significant to study the properties of equilibria of traffic network in different settings. Daniel et al (1999) studied the existence, characterization and computation of a special traffic network in a Banach space setting. Khanh and Luu (2004 & 2005) introduced the weak and strong Wardrop equilibria for multivalued cost functions and generalized the vector equilibrium principle to capacity constraint of paths. Recently, Lin (2010a & b) has developed some methods for equilibria with capacity constraints of arcs and (weak) vector equilibrium principle. Lin and Yu (2005) have also considered the related quasi-equilibrium problems. There are also more publications related to vector equilibrium flows of multiclass multicriteria traffic equilibria (Nagurney, 2000; Nagurney & Dong, 2002; Yang & Huang, 2002; Li & Chen, 2006). More references are available for this area and weighted equilibrium methods (Li et al, 2008; Yang & Goh, 1997; Browder, 1968; Fan, 1961). There are three important elements in the real traffic network: (1) multiple classes of vehicles, (2) multiple criteria, (3) constraints for capacity. All the models studied in the above papers or books lack at least one of the three elements. Therefore it is important to study traffic model with these three elements. We call it multiclass multicriteria traffic equilibrium problem with capacity constraint of arc. The multiclass multicriteria traffic equilibrium problem with capacity constrain of arc.

(MMTEPCCA) can be treated as a combination of the multiclass multicriteria traffic equilibrium problem and the multicriteria traffic problem with capacity constraint of arc.

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BACKGROUND

Before we talk about the equilibria let us recall some commonly used definitions and terminologies, which are referred to the references cited at the end of the paper. In a traffic network with multiclass we assume that there are m classes of vehicles (say, car, truck, bus, etc.) and set $\Omega=\{1,2,\ldots,m\}$. We use V to denote the set of all possible nodes on the traffic network, A the set of directed arcs, and Γ that of origin-destination (O-D) pairs. P_γ represents the collection of all available paths connecting O-D pair $\gamma\in\Gamma$ and $n=\sum_{\gamma\in\Gamma}|P_\gamma|$, namely the summation is over all $\gamma\in\Gamma$. Let $D=\left(d_\gamma^s\right)_{s\in\Omega,\gamma\in\Gamma}$ denote the demand vector, where $d_\lambda^s(>0)$ represents the traffic demand of class s of vehicles on O-D pair γ. For $a\in A$ and $s\in\Omega$ we use $f_a^s\left(s\in\Omega\right)$ to represent the flow of class s of vehicle on arc a, and arc flow is formulated as $f_a=\left(f_a^1,f_a^2,\ldots,f_a^m\right)^T\in R_+^m$. Let x_k^s stand for the traffic flow of class $s\in\Omega$ of vehicles on path $k\in P_\gamma$, then

$$\left(x_k^s\right)^T=\left(x_1^1,\ldots,x_n^1,x_1^2,\ldots,x_n^2,\ldots,x_1^m,\ldots,x_n^m\right)^T\in R_+^{nm}$$

is called a flow or path flow. If we define

$$\delta_{ak}=\begin{cases}1 & \left(a\in k\right)\\ 0 & \left(otherwise\right)\end{cases}$$

then $f_a^s=\sum_{\gamma\in\Gamma}\sum_{k\in P_\gamma}\delta_{ak}x_k^s$ and $f_a=f_a(x)$. For each $s\in\Omega$, let $\rho_s(>0)$ denote Passenger Car Unit (PCU) for class s of vehicles. $F_a=\sum_{s=1}^{m}\rho_s f_a^s$ represents the PCU flow for each $a\in A$. Assume $c_a(>0)$ denotes the capacity of PCU flow on arc a, then we call $C=\left(c_a\right)_{a\in A}$ the capacity vector. The system $\mathbb{T}=\left\{V,A,\Gamma,D,C\right\}$ is called a traffic network if V,A,Γ,D and C satisfy the conditions given above.

A flow x is called a feasible path flow or feasible flow if $\sum_{k\in P_\gamma}x_k^s=d_\gamma^s$ and $c_a\geq F_a\geq 0$ for each $a\in A$. We assume all the flows are feasible in this paper. We have the following lemma.

Lemma 1: Let

$$\vartheta=\left\{\begin{matrix}x\in R_+^{nm}:\forall s\in\Omega,\gamma\in\Gamma,\\ \sum_{k\in P_\gamma}x_k^s=d_\gamma^s\ and\,\forall a\in A,C_a\geq F_a\geq 0\end{matrix}\right\}$$

If we assume further that $\vartheta\neq\phi$ and d_γ^s is fixed for $s\in\Omega$ and $\gamma\in\Gamma$, then ϑ is convex and compact.

The proof the Lemma 1 is quite direct. Note we always assume that the conditions for Lemma 1 hold in the follow up network of traffic system. Let $Q_a^s=Q_a^s\left(f_a\right)=Q_a^s\left(x\right)\in R^{\tau_s}$ be the cost vector for vehicle class $s\in\Omega$ on an arc $a\in A$, where τ_s is a positive integer, then the cost vector for class s along path k is defined as $Q_k^s=\sum_{a\in A}\delta_{ak}Q_a^s\left(x\right)$, i.e. the sum over all the arc along k, for $\gamma\in\Gamma$ and $k\in P_\gamma$. We denote

$$Q^s\left(x\right)=\left(Q_1^s\left(x\right),Q_2^s\left(x\right),\ldots,Q_n^s\left(x\right)\right)=\begin{bmatrix}Q_{11}^s & Q_{21}^s & \cdots & Q_{n1}^s\\ Q_{12}^s & Q_{22}^s & \cdots & Q_{n2}^s\\ \cdots & L & \cdots & \cdots\\ Q_{1\tau_s}^s & Q_{2\tau_s}^s & \cdots & Q_{n\tau_s}^s\end{bmatrix}$$

and $Q(x)$ is defined as block diagonal matrix with all $Q^s(x)$ stacking together and the elements off the diagonal blocks are all zeros (see Box 1).

After the above definitions, the solution of equilibrium problem follows in the next section.

EQUILIBRIUM

An arc a is called a saturated arc of flow x if $F_a=C_a$; otherwise it is a non-saturated arc. $k\left(\in\bigcup_{\gamma\in\Gamma}P_\gamma\right)$ is said to be saturated path of flow