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Finite Automata Games: Basic Concepts

Fernando S. Oliveira
ESSEC Business School, Singapore

INTRODUCTION

Automata based systems have been used extensively in complex business modeling, for example, to represent Markov systems (e.g., Stewart et al., 1995; Uysal & Dayar, 1998; Gusak et al., 2003; Fuh & Yeh, 2001; Sbeity et al., 2008), in the development of classification systems (Gérard et al., 2005), in the analysis of commuters behavior (van Ackere & Larsen, 2004), to design electricity markets (Bunn & Oliveira, 2007, 2008), in the planning of real-options (Oliveira, 2010a), to study human-computer interaction (Gmytrasiewicz & Lisetti, 2002; Altuntas et al., 2007; Kim et al., 2010; Muller et al., 2013), to represent the relationship between emotions and reason (Oliveira, 2010c), in devising product differentiation strategies (Oliveira, 2010b), and in forecasting and production control (Liu et al., 2011).

The analysis of the behavior of such systems is very often based on the concepts of game theory, such as Nash equilibrium (e.g., Fudenberg & Tirole, 1991). The Nash equilibrium is a powerful tool for analyzing industries where there are strategic interdependences between players. However, it does not explain the process by which decision makers acquire equilibrium beliefs, failing to determine a unique equilibrium solution in many games, and, therefore, failing to predict, or prescribe, *rational behavior* (e.g., van Huyck et al., 1990; Samuelson, 1997; Fudenberg & Levine, 1998).

In games with multiple equilibria the Nash equilibrium fails to predict the players' behaviors. In this case, empirical studies (e.g., Roth & Erev, 1995) have shown that models of bounded rationality predict better than the Nash equilibrium does how people, organizations and markets behave (at least in the short run). A first attempt from the game theory literature to address this issue was to refine the concept of Nash equilibrium by including additional criteria. First, a player does not choose dominated strategies (Fudenberg & Tirole, 1991, p. 8). Second, choices in information sets not in the equilibrium path must be optimal choices (in order to avoid non-credible threats). This is called the rationalizability criterion (Bernheim, 1984; Pearce, 1984). However, the problem with equilibria selection still exists as different refinements select different equilibria. Furthermore, rationalizable strategies may be too demanding as they assume common knowledge of rationality.

Therefore, in order to model complex games, possibly with multiple equilibria, computer models which incorporate boundedly rational players are used as a mechanism for inductive equilibrium selection, and to test the validity of the perfect-rationality predictions. This methodological jump from perfect-rationality to bounded rationality has theoretical and philosophical implications. It corresponds to a switch from a "normative theory" to a "positive theory." The normative theory prescribes what each player in a game should do in order to promote his interests optimally (von Neumann & Morgenstern, 1953; van Damme, 1991, p. 1), whereas the positive theory describes how agents actually decide, as this line of research tries to understand how people and institutions behave (e.g., Samuelson, 1997, p. 3).

Simon (1972) was the first to emphasize the need to model bounded rationality in order to capture human and organizational behavior: see Sent (2004) for a review of Herbert Simon's work. As Aumann (1997) explains, people and organiza-

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tions use "rules of thumb" that they learned from experience when acting. In other words, people do not optimize even in simple decision problems, e.g., Salant (2011). This argument underlines the need to model the opponent's behavior, which was formalized in Rubinstein (1986, 1998) using finite automata - see Hopcroft & Ullman (1979) or Cecherini-Silberstein et al. (2012) for an introduction to automata theory. In order for inference of the opponents' strategic behavior to be possible some rules need to regulate the definition of strategies. Rubinstein (1986) proposed the finite automaton as a tool to model an agent's behavior. Salant (2011) has used automata to implement choice rules. An automaton is a decision rule, or a strategy, consisting of a finite set of states, a transition function (that defines the rules of transition between states) and a behavioral function (defining an agent's behavior in each state of the automaton). Rubenstein suggested that repeated games with finite automata could capture a player's bounded rationality (considering automata with a bounded number of states). At the same time, the introduction of finite automata constrains the type of strategies played: only regular strategies are admissible (i.e., given the same input, a player reacts always in the same manner). It is noteworthy, however, that long before Rubinstein had proposed the automata game, Schreider (1964) presented the formalism of dynamic programming to solve discrete deterministic problems using finite automata and introduced its possible application to game theory. In automata theory there are four major central issues: the complexity of computing the best response automaton, the equilibrium in automata games, automata inference and, finally, the dynamics problem.

THE FINITE AUTOMATA GAME

An automata game in the extensive form is a 5-tuple

$$G=\left(\mathrm{N},\ \left\{\mathrm{Z}^{\mathrm{i}}\right\}_{i=1}^{N},\ \left\{u^{i}\right\}_{i=1}^{N},\left\{Q^{i}\right\}_{i=1}^{N},\left\{\Sigma^{i}\right\}_{i=1}^{N}\right).$$

N denotes the number of players. Z^i represents a finite non-empty set of possible outcomes of the game, and each $z^i \in Z^i$ is a function of the actions of each player, $z^i = z\left(a^i, a^{-i}\right)$, where $a^i \in \Sigma^i$ represents an action of player i and $a^{-i} \in \Sigma^{-i}$ represents his opponents' actions. The outcomes of the game represent the information received by each player at the end of every stage. This information, or outcome, is a function of the actions of each player in the stage game, and it is different for each one of the players, as each one only knows the outcome of his own actions. $u^i = u\left(z^i\right)$ represents the utility function of player i, i.e., it is the payoff a player i perceives to have received from his action, given the perceived outcome. Q^i stands for a finite non-empty set of internal states of player i. Σ^i is a non-empty set of all possible actions of player i. The automata game G is an extensive form game where each player evolves a certain decision rule that may change at a certain iteration of the game. This decision rule, the automaton A^i, defines how a player reacts to the outcomes received from the environment.

A finite automaton used by the player i is a 5-tuple $A^i = \left(Q^i, q_0^i, \Sigma^i, \delta^i, \lambda^i\right)$ in which: Q^i is a finite non-empty set of internal states; q_0^i is the initial internal state; Σ^i is the set of all the possible actions; δ^i is a transition function $\left(\delta^i : Q^i \times Z^i \to Q^i\right)$ and λ^i is a behavioral function $\left(\lambda^i : Q^i \to \Sigma^i\right)$ associating an action to each possible internal state. At stage one each player i plays $\lambda^i\left(q_0^i\right)$. At a stage $t \geq 1$, after each player executing his actions with an outcome $z_t^i = \lambda^i\left(a^i, a^{-i}\right)$, each automaton A^i moves from the state q_t^i to the state $\delta^i\left(q_t^i, z_t^i\right)$. Then each player i chooses a new move $\lambda^i\left(q_{t+1}^i\right)$.

In an automata game, as Mor et al. (1996) put it, a player engages in three tasks at the same time: to define the strategy to play the game, to learn

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