

Utility Functions and Risk Attitudes in Decision Analysis

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INTRODUCTION

People and firms are faced with making decisions under uncertainty on a regular basis. These important decisions include making choices about insurance (health, automobile, etc.), medical treatments, and financial investments. Decision making would be simpler if everyone knew exactly whether they would get into an accident, have health problems, and have different investments perform well. Uncertainty exists because decisions are made before future outcomes are realized.

For the purpose of this article, the decision making process is analyzed in a quantifiable manner. Decisions are typically made without knowing future uncertain events. For example in a heads or tails gamble, a player does not know on which side the coin will land. In the game of roulette, the players do not know whether the ball will land on a red or a black number. Therefore, a logical decision maker would do mathematical calculations to find their expected wins/losses associated with each decision. In the game of a coin flip, it is easy to objectively quantify the chance event, since the probability of heads or tails occurring is 50%. But there are uncertain events where the probability of something occurring may require a more subjective judgment approach (Edwards, 1962; Anscombe & Aumann, 1963). For example, a person may want to determine whether to purchase health insurance. Then the task of assigning probabilities to whether one would become sick would be subjectively quantified.

We will introduce the three concepts of risk attitudes: risk-averse, risk-neutral and risk-seeking. To model risk attitudes for the decision making with uncertainty problem, we will apply the widely accepted Expected Utility Theory (Arrow, 1971; Friedman & Savage, 1948).

Research work has shown most people are in general risk-averse. They purchase insurance, invest in government bonds rather than stocks, etc. On the average, individuals spend more money on car insurance than the average amount car insurance companies pay out for each of their customers over a period of time. Otherwise, car insurance companies would be out of business. The insurance industry in America, which generates billions of dollars in revenues annually, is built upon the idea of risk-shifting. Drivers transfer the risk of getting into a costly accident to an insurance company. The insurance company earns a profit from their ability to engage in risk pooling (Arrow, 1996).

BACKGROUND

When a person considers whether to take a gamble, a typical first step is to calculate the *expected value* – which is the average amount one would win (or lose) if a gamble is played an infinite number of times. For example, let us consider an example in a fair coin heads or tails gamble where a player receives \$1000 dollars if the coin lands heads up and \$0 otherwise. Then the expected value of this gamble is

$$\frac{1}{2} \times \$1000 + \frac{1}{2} \times \$0 = \$500.$$

Let x_i represent the monetary gain (or loss) incurred with probability p_i of occurring.

The formula for computing the expected value for discrete gambles is:

$$ExpectedValue = \sum_{i=1}^{\infty} p_i x_i.$$

Referring back to the fair coin flip gamble with the possibility of gaining \$1000 on heads, and \$0 on tails, a *risk-neutral* person would be indifferent between receiving \$500 and taking the gamble. A *risk-averse* person would prefer receiving the \$500 with certainty to taking the gamble, while a *risk-seeking* person would prefer taking the gamble to receiving the \$500 with certainty. The three risk behaviors (risk-neutral, risk-averse, and risk-seeking) can be modeled with utility functions over all gamble amounts.

Consider the following gamble:

Which do you prefer?

A: \$500 for sure;

B: 50% chance to win \$0;
50% chance to win \$1000.

Research has shown that most people would choose choice A.

Expected Utility Theory states that lottery A is preferred to lottery B if and only if the expected utility of lottery A is greater than the expected utility of lottery B.

Expected Utility Theory enables us to explain why \$500 with certainty is preferred to a 50/50 gamble of \$0 or \$1000. Early decision scientists proposed that the utility function over monetary wealth is concave with respect to wealth (Pratt, 1964; Arrow, 1974).

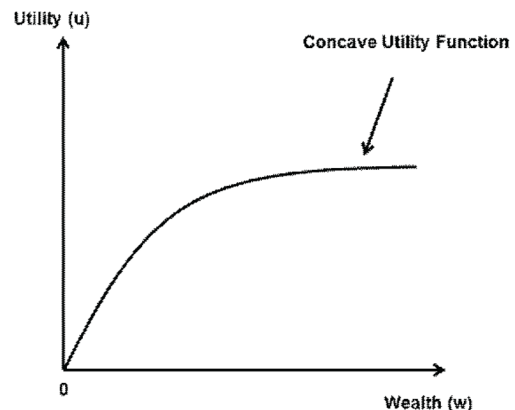
In Figure 1, the utility function is increasing and concave. The increasing function is consistent with the concept that more is always better (given

that wealth is a good thing). Concavity is reasonable since people in general have diminishing marginal utility of money. When your wealth is closer to the origin, every dollar is more critical to you, than when you already have thousands of dollars, that additional dollar would not give you as much marginal utility.

In choosing between choice A and B, the majority of people prefer the \$500 with certainty to a gamble with a 50/50 chance of either \$1000 or nothing. The expected value of choice B is \$500, which is equal to the expected value of Option A. But people in general are risk-averse, and so they should prefer Option A to Option B because the expected utility of choice B (as denoted by the projection of point B onto the vertical axis in Figure 2) is lower than the expected utility of choice A (as denoted by the projection of point A onto the vertical axis in Figure 2). Notice that in Figure 2, the Point D has the utility of initial wealth, $u(w_o)$, and similarly Point E has the utility of initial wealth plus \$1000, $u(w_o + 1000)$. Then Point B is the expected utility of a 50/50 gamble between gaining \$1000 and nothing,

$$.5u(w_o + 0) + .5u(w_o + 1000).$$

Figure 1. Utility versus wealth



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