Chapter 11
Path Relinking Scheme for the Max–Cut Problem

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ABSTRACT

In this chapter, a path relinking method for the maximum cut problem is investigated. The authors consider an implementation of the path-relinking, where it is utilized as a subroutine for another meta-heuristic search procedure. Particularly, the authors focus on the global equilibrium search method to provide a set of high quality solutions, the set that is used within the path relinking method. The computational experiment on a set of standard benchmark problems is provided to study the proposed approach. The authors show that when the size of the solution set that is passed to the path relinking procedure is too large, the resulting running times follow the restart distribution, which guarantees that an underlying algorithm can be accelerated by removing all of the accumulated data (set P) and re-initiating its execution after a certain number of elite solutions is obtained.

INTRODUCTION

The maximum cut problem is a well-known to be NP-hard (Karp, 1972), which recently gathered a lot of interest due to a number of important practical applications (Barahona, Grotschel, Junger & Reinelt, 1988; Chang & Du, 1988). The input for the maximum cut problem is an undirected graph $G = G(V,E)$, where each edge $(i, j) \in E$ is assigned a certain weight w_{ij}. Let (V_1, V_2) be a partition of the set of vertices V into two disjoint subsets. A cut (V_1, V_2) in G is any subset of edges $(i, j) \in E$, such that $i \in V_1$ and $j \in V_2$. The maximum cut problem consists in finding a cut in graph G with the maximum sum of the edge weights.

In the current chapter we describe an extension of the algorithm for the maximum cut problem based on the global equilibrium search (GES) method. Earlier comparison with available algorithms using a set of benchmark problems show that GES performs favorably compared to other approaches in terms of computational speed and

solution quality (Shylo & Shylo, 2010). The GES method maintains a set of solutions, which are used to prevent algorithm from converging to previously visited areas in the search space. Since this set contains high quality solutions, it is desirable to utilize this set more efficiently, instead of using it just to prevent visiting the same search space areas. Assuming that these high quality solutions share some common structure, one can combine their components in an attempt to find an enhanced solution, or restrict the search to some subset of solutions defined by them. In the current chapter, we describe a scheme where a path relinking procedure is embedded within the GES method to provide such functionality.

BACKGROUND

Path relinking method searches for solutions of an optimization problem along the trajectories that connect solutions from a given set (Glover, Laguna & Marti, 2000). The most common path relinking scheme involves a pair of solutions: an initiating solution and a guiding solution. A set of moves (transformations) are applied starting in the initiating solution that sequentially introduce the attributes of the guiding solution. Usually, such moves result in a sequence of solutions that lie on a path between the initial solution pair.

Assuming that the weights of the graph edges are non-negative, the maximum cut problem can be formulated by the following mixed-integer program (Kahruman, Kolotoglu, Butenko & Hicks, 2007):

$$\sum_{i,j=1,i<j}^{n} w_{ij} y_{ij}$$
$$s.t. \quad y_{ij} - x_i - x_j \leq 0 \quad i,j = 1,...,n, \quad i < j$$
$$y_{ij} + x_i + x_j \leq 2 \quad i,j = 1,...,n, \quad i < j$$
$$x \in \{0,1\}^n$$

The optimal solution vector x defines a graph partition $\{V_1, V_2\}$ (if $x_i = 1$ then $v_i \in V_1$, otherwise $v_i \in V_2$) that has the maximum cut value, the sum of weight of edges connecting different partitions. Let $f(x)$ denote a cost of a cut corresponding to the solution vector x.

Local search based methods for the max-cut problem require an initial solution $x \in \{0,1\}^n$ to start the chain of local improvements until the local optimum is obtained. GES provides an intelligent mechanism of generating initial solutions for local search based methods and it proved to be efficient for a variety of combinatorial problems (Pardalos, Prokopyev, Shylo & Shylo, 2008; Shylo, Prokopyev & Shylo, 2008). The variation of the global equilibrium search for the Max-Cut problem that does not depend on the path-relinking methodology can be found in (Shylo et al, 2012).

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The generation probabilities in GES are defined by some subset S of previously visited solutions (e.g., a set of local optima). These probabilities are parameterized by an ordered set of temperature values $0 \leq \mu_0 < \mu_1 < ... < \mu_K$, which bear the same function as a cooling schedule in the simulated annealing method (Aarts *& Korst, 1989)*. The search process is organized as a repeating sequence of K temperature stages, one for each temperature value. A fixed number of initial solutions are generated at each temperature stage to be used as starting solutions for some local search based method. In case of binary decision variables, the generation procedure at temperature stage k sets jth component to 1 (or 0) with probability given by $p_j(\mu_k)(1 - p_j(\mu_k))$:

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