Comparing Conventional and Non–Parametric Option Pricing

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INTRODUCTION

Once, the seminal Black–Scholes (Black & Scholes, 1973) model was thought to be the last word on option pricing: all that was needed, it was thought, was some adjustments and it could be applied to price options on any financial instrument.

In the past two decades, increases in the bias of these Black–Scholes style models (Rubinstein, 1985) have led researchers to develop new models, coined modern parametric option pricing models. The underlying logic of this modern parametric option pricing program is that by loosening the "unrealistic" assumptions of the conventional Black–Scholes style option pricing models (e.g., including jumps in stochastic processes describing underlying asset prices (Merton, 1973); incorporating skewness and/or kurtosis describing the underlying asset distribution (Corrado & Su, 1996); considering the effects on demand and/or supply (Follmer & Sondermann, 1986); and the effects of tax (Scholes, 1976)), it is possible to mitigate the bias associated with them.

However, recently, many authors (e.g., Bakshi, Cao & Chen, 1997; Bakshi & Chen, 1997) have found that the modern parametric models are found to:

- be too complex;
- have poor out of sample performance;
- have implausible/inconsistent implied parameters.

Perhaps the final word on modern parametric models should be left to Fischer Black (Black, 1998): "The Black–Scholes formula is still around, even though it depends on at least 10 unrealistic assumptions. Making the assumptions more realistic hasn't produced a formula that works better across a wide range of circumstances".

FUTURE TRENDS: NON-PARAMETRIC OPTION PRICING

If we concede that the modern parametric option pricing program has failed, where does this leave us? There remains significant, persistent and systematic bias in modern parametric option pricing models (Rubinstein, 1985). The ideal option pricing model would not only provide unbiased option prices, be consistent with the underlying process and distributions of asset returns, and have minimal assumptions and parameters to estimate, it would also incorporate a statistical estimate of option pricing error. This ideal model may never be found by generalizing the unrealistic assumptions of the conventional option pricing models. What alternative is left?

Either we must accommodate option pricing model error explicitly and surrender any notion of improvement, or some new, alternative approach must be used. Non-parametric techniques represent such an alternative approach.

There are three scenarios where non-parametric approaches are particularly useful:

- when conventional/modern parametric solutions lead to bias in pricing;
- when conventional/modern parametric solutions exist but are too complex to use;
- and, when conventional/modern parametric solutions do not exist.

Non-parametric option pricing models can be divided into two separate strands: the model free and the semi-parametric. These two strands can be characterized by their dependence on finance theory. Model free non-parametric techniques have no reliance on finance theory whatsoever; semi-parametric approaches have a high reliance on finance theory (e.g., approaches that augment parametric option pricing models with non- parametric techniques).

Numerous technologies have been used to estimate model free non-parametric option pricing, including: genetic programming (Chen, Yeh & Lee, 1998), kernel regression (Ait-Sahalia & Lo, 1995; Broadie, Detemple, Ghysels & Torres, 1996) and artificial neural networks (Malliaris & Salchenberger, 1993).

The fundamental problem of non-parametric model free option pricing models lies with their greatest strength: their independence from the assumptions of finance theory. On the one hand, this independence is a great strength because being free of these assumptions means that the persistent, systematic and significant bias found in parametric option prices may be eliminated. On the other hand, this independence is a great weakness. The fact that non-parametric model free option pricing approaches do not rely on any finance theory for their derivation means that there is no guarantee that the prices obtained from these models will not conform to rational pricing[1].

So, the cost of the complete flexibility of the model free non-parametric option pricing approaches is that there can be no guarantee of rational pricing. Ghysels et al. (Ghysels, Patilea, Renault & Torres, 1997) state:

"non parametric model free option pricing becomes quickly infeasible since it is not able to capture a large set of crucial restrictions implied by arbitrage."

The aim of semi-parametric option pricing models is to estimate a portion of the option pricing model non-parametrically while retaining the conventional option pricing model framework to guarantee rational pricing.

There are three main branches of semi-parametric option pricing. Hybrid approaches model conventional option pricing residual error in the hope of reducing this error (Lajbcygier, 2003). General volatility models use non- parametric techniques to estimate the volatility used in conventional parametric option pricing (Dumas, Fleming & Whaley, 1996). Finally, equivalent martingale measures (or risk neutral pricing) (Campbell, Lo & Mackinlay, 1997) use non- parametric techniques to estimate the risk neutral probability distribution, which in turn can be used to price options.

CRITICAL ISSUES

Despite all its promise, non-parametric option pricing approaches are considered an emerging technology with the potential to help improve option pricing. There are still many open critical issues (some of which are discussed in Table 1).

CONCLUSION

The ideal option pricing model would not only provide unbiased option prices, be consistent with the underlying process and distributions, and have minimal assumptions and parameters to estimate, it would also incorporate a statistical estimate of option pricing error. No such model exists today. It is interesting to speculate as to whether such a model will exist in the future, and if the model will be parametric, non-parametric, or a mixture of the two.

As Campbell, Lo and Mackinlay (1997) have stated, non-parametric option pricing approaches hold promise:

"Although it is still too early to tell if these non parametric and highly data intensive methods will offer improvements over their parametric counterparts, the preliminary evidence is quite promising."

REFERENCES

Ait-Sahalia, Y., & Lo, A.W. (1995). *Nonparametric estimation of state-price densities implicit in financial asset prices.* Working Paper No. LFE-1024-95. MIT-Sloan School of Management, Laboratory for Financial Engineering.

Table 1. Summary of critical issues of non- parametric option pricing

Elimination of Option Pricing Bias
To eliminate persistent, systematic and significant bias in option pricing is a critical requirement of any model.

Poor out of sample fit of modern parametric option pricing models
The fact that many modern parametric models provide inaccurate pricing out of sample (i.e., for data which were not used for parameter estimation) presents a major flaw.

Implausible Implied Parameters
Some parameters implied by a modern parametric model fitted to a data set (e.g., skewness) may be nonsensical.

Arbitrage and Option Pricing
The process by which "riskless profit" can be earned when certain pricing conditions do not hold.