

# Chapter 30

## Solving Non-Linear Bi-Level Programming Problem Using Taylor Algorithm

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### ABSTRACT

*In recent years the bi-level programming problem (BLPP) is interested by many researchers and it is known as an appropriate tool to solve the real problems in several areas such as economic, traffic, finance, management and so on. Also it has been proved that the general BLPP is an NP-hard problem. The literature shows a few attempts for using approximate methods. In this chapter we attempt to develop an effective approach based on Taylor theorem to obtain an approximate solution for the non-linear BLPP. In this approach using the Karush-Kuhn-Tucker, the BLPP has been converted to a non-smooth single problem, and then it is smoothed by the Fischer – Burmeister function. Finally the smoothed problem is solved using an approach based on Taylor theorem. The presented approach achieves an efficient and feasible solution in an appropriate time which is evaluated by comparing to references and test problems.*

### 1. INTRODUCTION

The bi-level programming problem (BLPP) is a nested optimization problem, which has two levels in hierarchy. The first level is called leader and the second one is called follower. They have their own objective functions and constraints. The leader actions first, and the follower reacts to the leader decision. The follower should optimize its objective function according to the leader decision and delivered answers of the leader. In fact, the leader inflicts his decision on and obtains reaction of the follower.

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It has been proved that the BLPP is an NP- Hard problem even to seek for the locally optimal solutions (Bard, 1991; Vicente et.al, 1994). Nonetheless the BLPP is an applicable problem and a practical tool to solve decision making problems. It is used in several areas such as transportation, finance and so on. Therefore finding the optimal solution has a special importance to researchers.

Several algorithms have been presented for solving the BLPP (Lv, et. al., 2007; Allende, et. al, 2013; Mathieu, 1994; Wang, et. al., 2003; Weng, et. Al, 2000; Bard, 1991; Jiang, et. al. 2015). These algorithms are divided into the following classes: Transformation methods (Lv, et. al. 2007; Allende, et. al, 2013; Brotcorne, et. al, 2013; Dempe, et. al, 2012), Fuzzy methods (Sakawa, et. al. 1998; Shih, et. al. 1996; Pramanik, et. al. 2007; Sakawa, et. al. 2001; sakawa, et. al, 2012), Global techniques (Nocedal, 2006; Khayyal, 1985; Mathieu, 1994; Wang, et. al. 2003), Primal–dual interior methods (Weng, et. Al, 2000), Enumeration methods, (Thoai, et. al, 2004), Meta heuristic approaches (Hejazi, et. al, 2002; Wang, et. al, 2008; Hu, et. al, 2010; Pal, et. al. 2010; Wan, et. al. 2013, 25]. In the remaining of section, two classes of methods have been explained.

## **1.1 Enumeration Methods**

Branch and bound is an optimization algorithm that uses the basic enumeration. But in these methods we employ clever techniques for calculating upper bounds and lower bounds on the objective function by reducing the number of search steps. In these methods, the main idea is that the vertex points of achievable domain for BLPP are basic feasible solutions of the problem and optimal solution is among them.

## **1.2 Transformation Methods**

An important class of methods for constrained optimization seeks the solution by replacing the original constrained problem with a sequence of unconstrained sub-problems or a problem with simple constraints. These methods are interested by some researchers for solving BLPP, so that they transform the follower problem by methods such as penalty functions, barrier functions, Lagrangian relaxation method or KKT conditions. In fact these techniques convert the BLPP into a single problem and then it is solved by other methods.

In this chapter, we propose a procedure based on Taylor theorem and consider the linear bi-level programming problem. In the remaining of pages, in section 2 basic concepts and definitions of the BLPP are proposed. We provide main theoretical results in section 3. Section 4 describes the steps of the presented algorithm. Computational results are proposed in Section 5 and finally, the chapter is finished in section 6.

## **2. THE NON-LINEAR BLPP AND SMOOTHING METHOD**

The BLPP is used frequently by problems with decentralized planning structure. It is defined as (Bard, 1999):

$$\begin{aligned} & \min_x F(x, y) \\ & \text{s.t } \min_y f(x, y) \\ & \text{s.t } g(x, y) \leq 0, \quad x, y \geq 0. \end{aligned}$$

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