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Optimizing Paths with Random Parameter Distributions

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INTRODUCTION

This research is grounded in the view that organizations are information processing systems. Organizations design their structure, processes, and information technologies for the purpose of processing, exchanging, and distributing the information required for their functions. The volume of information exchanged is not always the same. Thus in order to provide an efficient process of communication we propose an algorithm which determines the path that minimizes the expected value of an utility function over a dynamic probabilistic network with discrete or continuous real random variables (parameters) associated to each emerging arc. To obtain the optimal dynamic path from a source to sink node in the discrete case, we use a generalization of Bellman first-in-first-out labeling correcting algorithm used to determine the shortest path in directed networks with deterministic parameters associated to each arc. In the case where arc parameters are continuous random variables we propose algorithms involving multi-objective optimization. Additionally, some initialization techniques that improve the running times without jeopardizing memory are also considered. The topology of the networks is not known in advance, which means that we only have knowledge of the incoming (outgoing) arcs, and their parameters, of some specific node once we reach it. Thus the optimal path is determined in a dynamic way. We also present computational results for networks with 100 up to 10,000 nodes and densities 2, 5, and 10.

Suppose we want to obtain the optimal path in a directed random network, where the parameters associated to the arcs are real random variables following discrete or continuous distributions. The criterion that has been chosen to decide which path is optimal is the one that minimizes the expected value of an utility function over the considered network.

This methodology can be used in different applications, as energy network or data network, where real on-time optimal solutions are necessary. Different types of optimal path problems over random probabilistic networks are considered in the literature. Considering networks with random parameters that are all realized at once, the first publication belongs to Frank (1969) who determined the shortest path on a random graph and presented a process for obtaining the probability distribution of the shortest path.

In 1991, Bard and Bennett (1991) developed heuristic methods based on Monte-Carlo simulations for the stochastic optimal path with non-increasing utility function. Their computational results regarded networks with 20 up to 60 nodes. For the special case of acyclic networks, Cheung and Muralidharan (1995) developed a polynomial time algorithm (in terms of the number of realizations per arc cost and the number of emerging arcs per node) to compute the expected cost of the dynamic stochastic shortest path.

In 2004, Rasteiro and Anjo (2004) developed an algorithm to solve the stochastic optimal path problem considering random continuous parameters associated to the network arcs.

In this paper we propose an algorithm which determines the path that minimizes the expected value of an utility function over a dynamic probabilistic network with discrete or continuous real random variables (parameters) associated to each emerging arc. If the variables are discrete, our algorithm is a generalization of Bellman first-in-first-out labeling-correcting algorithm used to determine the shortest path in directed networks with deterministic parameters associated to each arc.

The most common methods used to solve the classical deterministic shortest path problem are the label-correcting methods. In this approach to each node $i \in N$, ((N, A) is a probabilistic network where $N = \{v_1, \ldots, v_n\}$ is the set of nodes and $A = \{a_1, \ldots, a_m\} \subseteq N \times N$ is the set of arcs) is assigned a label (usually the distance or cost to the destination node) and put into a queue. Then we scan through the nodes in the queue and update the labels, if necessary. After a node is scanned, it will be removed from the queue, and then some nodes may be inserted or repositioned in the queue. The process is repeated until the queue is empty or all labels are correct. There are several implementations of these methods including the first-in-first-out algorithm of Bellman (1958). A review of these methods, which basic difference is the way how the queue is manipulated, can be found in Ahuja, Magnanti, and Orlin (1993).

We present computational results, for networks with 100 up to 10,000 nodes and densities 2, 5, and 10, which prove that our approach is very efficient in terms of memory and time.

If the variables are continuous, we will concentrate on the linear, quadratic, and exponential utility functions presenting a theoretical formulation based on multi-criteria models as well as the resulting algorithms and computational tests.

This paper is divided in two parts. The first part refers to theory and results obtained for discrete random variables, and the second one is dedicated to the continuous case.

PROBLEM DEFINITION: DISCRETE CASE

In the stochastic shortest path problem, a directed probabilistic network (N, A) is given where each arc $(i, j) \in A$ is associated to the real random variable X_{ij} which is called the random parameter of the arc $(i, j) \in A$. We assume that the real random variables X_{ij} have discrete distributions and are independent. The variables X_{ij} are sometimes referred as cost, time, or distance.

The set of outcomes of X_{ij} will be denoted by $S_{X_{ij}} = \{d_{ij}^1, \cdots, d_{ij}^r\}$. We will assume that the dimension of $S_{X_{ij}}$, that is, r is always a finite value. The probability of X_{ij} assume the value d_{ij}^l is denoted by p_{ij}^l. If an appropriate utility measure is assigned to each possible consequence and the expected value of the utility measure of each alternative is calculated, then the best action is to consider the alternative with the highest expected utility (which can be the smallest expected value). The choice of an adequate utility function for a specific type of problem can be taken using direct methods presented in Pratt et al. (1965).

The utility of the arc $(i, j) \in A$ in the optimal path is measured calculating the minimum of the real random variables X_{iw} where w is such that the arc $(i, w) \in A$, that is, w belongs to the forward star of i. The forward star of node i is the set formed by the terminal nodes of its outgoing arcs. Thus $U((i, j)) = \min_{w \in F(i)} X_{iw}$.

Associated to the path p, we define the real random variable $X_p = \sum_{(i,j)\in p} \min_{w\in F(i)} X_{iw}$ representing the random parameter of the loopless path $p \in P$.

With the objective of determining the optimal path, we consider a real function $U : P \to \Re$, called *utility function*, such that for each loopless path p, $U(p)$ depends on the random variables associated to the arcs of p and is defined as

$$U(p) = \mathrm{E}\left(\sum_{(i,j)\in p} \min_{w\in F(i)} X_{iw} \right).$$

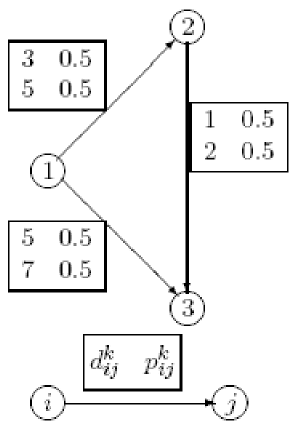
In the dynamic stochastic shortest path, we want to determine the loopless path $p^* \in P$ that minimizes the expected value of the utility function. The loopless path p^* is called *optimal solution* of the referred problem.

The problem can then be mathematically defined as

$$\begin{aligned} \min_{p\in P} U(p) &= \min_{p\in P} \mathrm{E}\left(\sum_{(i,j)\in A} \min_{w\in F(i)} X_{iw} \right) \\ &= \min \quad \mathrm{E}\left(\sum_{(i,j)\in A} \min_{w\in F(i)} X_{iw} \right) Y_{ij} \\ \text{s.t.} & \\ \sum_{\substack{(i,j)\in A \\ Y_{ij}\in\{0,1\}}} Y_{ij} - Y_{ji} &= \begin{cases} 1, & i = s \\ 0, & i \notin \{s,t\} \\ -1, & i = t \end{cases} \end{aligned} \quad (1)$$

Since the constraint matrix is totally unimodular,

Figure 1. Network example



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