Chapter 2
Methodologies for Learning and/or Teaching

ABSTRACT

In this chapter, there is an introduction of the problem of methodologies and strategies for learning and/or teaching mathematics in primary and secondary school. The first methods are the symbolic-reconstructive and perceptive-motor. For the authors, the problem about contemporary teaching is characterized by the relationship between mathematics and electronic technology. Finally, a new didactic approach will be proposed. It is called "from concrete/virtual-concrete to abstract" and it consists in the introduction of a new computer-based phase, called graphic-numerical, in a good traditional didactic path.

"He who does not know how to teach the things he knows, knows for himself, not for others; he must be considered as one who knows nothing" ~Pietro Abelardo (1079-1142)

"You understood why you managed to make someone understand" ~Henrie Freud (1823-1915)

1. INTRODUCTION

In the previous chapter, we have tried to highlight some aspects of mathematical thinking, also outlining its, sometimes radical, evolution from the nineteenth century onwards. Now, we want to deal with learning supported by teaching in the places provided for it. It is redundant to state that the mathematical discipline has been imposed for the needs of social life from its earliest years of life. It is important, instead, to emphasize that it is indispensable for acquiring culture, in the broad sense of the term, for the formation of thought in all its components (rational, intuitive, creative, etc.) or in a word, it is "productive" (Werthmeier, 1959). The learning environment, in particular the teacher, the classroom-laboratory etc., is of fundamental importance in cultural education. Primarily, the teacher's attitude in regards of

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both social, in this case the student as person, and mathematics is of fundamental importance. We believe that it is indispensable to have a deep human respect, almost "a love", for young people. Indeed, we fully agree with Hans Freudenthal:(Freudenthal, 2006) "The value that you attribute to the students as human beings determines the way in which they learn their mathematics: with freedom or as slaves, guided or harnessed. Regarding mathematics, it is necessary to be cultured and have a correct and profound mastery of the discipline. ." And also "(...) certainty must be sought and guaranteed, and in mathematics this is achieved with a very particular mental activity. And it is this mental activity, rather than the contents, that characterizes mathematics as the field in which it can be exercised in the most appropriate and efficient way ". (Freudenthal, 2006) In fact, if the teacher thinks that mathematics is a "basket of formulas" from which we can take in a disordered manner, with rules that appear almost magical, at best we can expect that the students acquire only a formal training, memorized for a short time. On the contrary mathematics is a search for certainty, it is a mental activity that is expressed in the construction of structures, through which our mind "reads" (so to speak) the reality that surrounds us.

The learning of mathematics, therefore, must be substantial appropriation of a certain attitude, rather than an accumulation of formal notions. The laboratory-classroom, to some extent, is affected by the degree of civilization and social development, i.e. it is strongly linked to the historical era to which it refers. For example, today's high technological level cannot fail to influence the intellectual and cultural formation of today's young people. For our part in this context, this means that the existing technology influences the cultural and, especially, mathematical education for young people. In other words, the digital computer becomes a new partner in the "didactic contract".(Balacheff & Kaput, 1996) As we will see in more detail later, electronic machines influence mathematical thought and, consequently, the teaching of mathematics. The teaching methods can be of various and the educational choices depend largely on the cultural and educational level of the individual teacher. Certainly, we cannot here review the various didactic approaches, but we must limit ourselves to outlining some, before setting out our convictions. It is based on two fundamental aspects of modern mathematical thought, which we discussed in the previous chapter. Both of them are linked to the particular degree of socio-cultural and technological development of today. We intend to refer to the artificial, symbolic, abstract, etc language, and to the connection between reality and mathematical doctrine.

2. MATHEMATICAL LANGUAGE: APPROACH STRATEGIES.

As we have repeatedly emphasized, the mathematical discipline is expressed by a formal and artificial language, with its own grammatical and syntactic rules. It pervades, stimulates and represents a large part of scientific knowledge. It is natural, therefore, to ask oneself which didactic strategies are more appropriate to use to present to students this abstract mathematical formalism through a reasonable degree of concreteness, in order to make it as interesting and real as possible. It is very probable, in fact, that a direct approach of mathematical symbolism, with its rigid rules of manipulation, without any justification and connection with concrete models, cannot arouse great interest on the part of the majority of the students. An abstract and general language, able to interpret different situations, acquires flavour and interest only when experience can provide us with numerous examples of concrete cases as real situations that are different from each other, but suitable to be represented with calculation algorithms.

An example, in this regard, as simple as it is illuminating is constituted by natural or even relative numbers. In fact, apart from the definition of numbers, it is sufficient to observe that the set of natural