Chapter 14
Fractals, Computer Science and Beyond

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ABSTRACT

In the modelling of the natural shapes (clouds, ferns, trees, shells, rivers, mountains), the limits imposed by Euclidean geometry can be exceeded by the fractals. Fractal geometry is relatively young (the first studies are the works by the French mathematicians Pierre Fatou (1878-1929) and Gaston Julia (1893-1978) at the beginning of the 20th century), but only with the mathematical power of computers has it become possible to realize connections between fractal geometry and the other disciplines. It is applied in various fields now, from the biology to the architecture. Important applications also appear in computer science, because the fractal geometry permits to compress the images; to reproduce, in the virtual reality environments, the complex patterns and the irregular forms present in nature using simple iterative algorithms execute by computers. Recent studies apply this geometry for controlling the traffic in the computer networks (LANs, MANs, WANs, and the Internet) and in the realization of virtual worlds based on World Wide Web. The aim of this chapter is to present fractal geometry, its properties (e.g., the self similarity), and their applications in computer science (starting from the computer graphics, to the virtual reality).

INTRODUCTION

Fractal geometry is a recent discovery. It is also known as "Mandelbrot's geometry" in honour to its "father" the Polish-born Franco-American mathematician Benoit Mandelbrot, that showed how fractals can occur in many different places in both mathematics and elsewhere in nature.

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Fractal geometry is now recognized as the true Geometry of Nature. Before Mandelbrot, mathematicians believed that most of the patterns of nature were far too irregular, complex and fragmented to be described mathematically. Mandelbrot's geometry replaces Euclidian geometry which had dominated our mathematical thinking for thousands of years.

The Encyclopaedia Britannica introduces the fractal geometry as follow (2007): "In mathematics, the study of complex shapes with the property of self-similarity, known as fractals. Rather like holograms that store the entire image in each part of the image, any part of a fractal can be repeatedly magnified, with each magnification resembling all or part of the original fractal. This phenomenon can be seen in objects like snowflakes and tree bark… This new system of geometry has had a significant impact on such diverse fields as physical chemistry, physiology, and fluid mechanics; fractals can describe irregularly shaped objects or spatially nonuniform phenomena that cannot be described by Euclidean geometry".

The multiplicity of the application fields had a central role in the diffusion of fractal geometry (Mandelbrot, 1982; Nonnenmacker et al., 1994; Eglash, 1999; Barnsley et al., 2002; Sala, 2004; Sala, 2006; Sala, 2008; Vyzantiadou et al., 2007).

BACKGROUND: WHAT IS A FRACTAL?

A fractal could be defined as a rough or fragmented geometric shape that can be subdivided in parts, each of which is approximately a reduced-size copy of the whole (Mandelbrot, 1988).

"Fractal" is a term coined by Benoit Mandelbrot (b. 1924) to denote the geometry of nature, which traces inherent order in chaotic shapes and processes. The term derived from the Latin verb "frangere", "to break", and from the related adjective "fractus", "fragmented and irregular". This term was created to differentiate pure geometric figures from other types of figures that defy such simple classification. The acceptance of the word "fractal" was dated in 1975. When Mandelbrot presented the list of publications between 1951 and 1975, date when the French version of his book was published, the people were surprised by the variety of the studied fields: linguistics, cosmology, economy, games theory, turbulence, noise on telephone lines (Mandelbrot, 1975). Fractals are generally self-similar on multiple scales. So, all fractals have a built-in form of iteration or recursion. Sometimes the recursion is visible in how the fractal is constructed. For example, Koch snowflake, Cantor set and Sierpinski triangle are generated using simple recursive rules. The self similarity, the Iterated Function Systems and the Lindenmayer System are applied in different fields of computer science (e.g., in computer graphics, in virtual reality, and in the traffic control of computer networks).

The Self-Similarity

The self-similarity, or invariance against changes in scale or size, is a property by which an object contains smaller copies of itself at arbitrary scales. Mandelbrot defined the self-similarity as follow:" When each piece of a shape is geometrically similar to the whole, both the shape and the cascade that generate it are called self-similar" (Mandelbrot, 1982, p. 34).

A fractal object is self-similar if it has undergone a transformation whereby the dimensions of the structure were all modified by the same scaling factor. The new shape may be smaller, larger, translated, and/or rotated. "Similar" means that the relative proportions of the shapes' sides and internal angles remain the same. As described by Mandelbrot (1982), this property is ubiquitous in the natural world. Oppenheimer (1986) used the term "fractal" exchanging it with self-similarity, and he affirmed that the geometric notion of self-similarity is evolving in a paradigm for modelling the natural world, in particular in the world of botany.

Self-similarity appears in objects as diverse as leaves, mountain ranges, clouds, and galaxies. Figure 1a shows a snowflake which is an example of self-similarity in the nature. Figure 1b illustrates a Koch snowflake, it is built starting from an equilateral triangle, removing the inner third of each side, building another equilateral triangle at the location where the side was removed, and then repeating the process indefinitely. This