Chapter 3
Path Relinking Scheme for the Max-Cut Problem within Global Equilibrium Search

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ABSTRACT

In this paper, the potential of the path relinking method for the maximum cut problem is investigated. This method is embedded within global equilibrium search to utilize the set of high quality solutions provided by the latter. The computational experiment on a set of standard benchmark problems is provided to study the proposed approach. The empirical experiments reveal that the large sizes of the elite set lead to restart distribution of the running times, i.e., the algorithm can be accelerated by simply removing all of the accumulated data (set P) and re-initiating its execution after a certain number of elite solutions is obtained.

PATH RELINKING SCHEME FOR THE MAX-CUT PROBLEM WITHIN GLOBAL EQUILIBRIUM SEARCH

The maximum cut problem is a well-known NP-hard problem (Karp, 1972), which recently gathered a lot of interest due to a number of important practical applications (Barahona, Grotschel, Junger, & Reinelt, 1988; Chang & Du, 1988). The input for the maximum cut problem is an undirected graph $G = G(V,E)$, where each edge $(i,j) \in E$ is assigned a certain weight w_{ij}. Let (V_1, V_2) be a partition of the set of vertices V into two disjoint subsets. A cut (V_1, V_2) in G is any subset of edges $(i,j) \in E,$ such that $i \in V_1$ and $j \in V_2$. The maximum cut problem is to find a cut in graph G with the maximum sum of the edge weights.

In the current paper we consider an extension of the algorithm for the maximum cut problem based on global equilibrium search (GES). The

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comparison with other available algorithms using a set of benchmark problems revealed that GES dominates other approaches in terms of computational speed and solution quality (Shylo & Shylo, 2010). The implementation of GES maintains a set of solutions, which are used to prevent algorithm from converging to previously visited areas in the search space. Since this set contains high quality solutions, it is desirable to use it in a more efficient manner. Assuming that high quality solutions share some common structure, one can try to combine their components in an attempt to find an enhanced solution. In the current paper, we propose an extension of path relinking to GES to achieve this goal.

Path relinking method organizes a search for an improvement based on some population of solutions (Glover, Laguna, & Marti, 2000). The most common path relinking scheme involves a pair of solutions: an initiating solution and a guiding solution. A set of moves (transformations) are applied starting in the initiating solution that sequentially introduce the attributes of the guiding solution. Usually, such moves results in a set of solutions that lie on a path in the search space between the initial solution pair.

METHOD

Assuming that the weights are non-negative, the maximum cut problem can be formulated by the following mixed-integer program (Kahruman, Kolotoglu, Butenko, & Hicks, 2007):

$$\sum_{i,j=1,i<j}^{n} w_{ij} y_{ij}$$

$$\begin{aligned} s.t.\quad & y_{ij} - x_i - x_j \leq 0 \quad i,j = 1,\dots,n,\ \ i<j \\ & y_{ij} + x_i + x_j \leq 2 \quad i,j = 1,\dots,n,\ \ i<j \\ & x \in \{0,1\}^n \end{aligned}$$

The optimal solution vector *x* defines a graph partition $\{V_1, V_2\}$ (if $x_i = 1$ then $v_i \in V_1$, otherwise $v_i \in V_2$) that has the maximum cut value. Let $f(x)$ denote a cost of a cut corresponding to the solution vector *x*.

Local search based methods require an initial solution $x \in \{0,1\}^n$ to start the chain of local improvements until the local optimum is obtained. GES provides an intelligent mechanism of generating initial solutions for local search based methods. Its metaheuristic framework proved to be extremely efficient for a variety of combinatorial problems (Pardalos, Prokopyev, Shylo, & Shylo, 2008; Shylo, Prokopyev, & Shylo, 2008).

The generation probabilities in GES are defined by some subset *S* of previously visited solutions (e.g., a set of local optima). These probabilities are parameterized by an ordered set of temperature values $0 \leq \mu_0 < \mu_1 < \dots < \mu_K$, which bear the same function as a cooling schedule in the simulated annealing method (Aarts & Korst, 1989). The search process is organized as a repeating sequence of *K* temperature stages, one for each temperature value. A fixed number of initial solutions are generated at each temperature stage to be used as starting solutions for some local search based method. In case of binary decision variables, the generation procedure at temperature stage *k* sets *j*th component to 1 (or 0) with probability given by $p_j(\mu_k)(1 - p_j(\mu_k))$:

$$p_j(\mu_k) = \left(1 + \exp\{\sum_{i=0}^{k-1} \frac{\mu_{i+1} - \mu_i}{2}(E_{ij}^0 + E_{i+1j}^0 - E_{ij}^1 - E_{i+1j}^1)\}\right)^{-1} \quad (1)$$

where $E_{kj}^1 \left(E_{kj}^0\right)$ is a weighted sum of objective values, that corresponds to solutions in *S*, such that $x_j = 1$ $\left(x_j = 0\right)$: