

Chapter 11

Inference Algebra (IA): A Denotational Mathematics for Cognitive Computing and Machine Reasoning (I)

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ABSTRACT

Inference as the basic mechanism of thought is one of the gifted abilities of human beings. It is recognized that a coherent theory and mathematical means are needed for dealing with formal causal inferences. This paper presents a novel denotational mathematical means for formal inferences known as Inference Algebra (IA). IA is structured as a set of algebraic operators on a set of formal causations. The taxonomy and framework of formal causal inferences of IA are explored in three categories: (a) Logical inferences on Boolean, fuzzy, and general logic causations; (b) Analytic inferences on general functional, correlative, linear regression, and nonlinear regression causations; and (c) Hybrid inferences on qualification and quantification causations. IA introduces a calculus of discrete causal differential and formal models of causations; based on them nine algebraic inference operators of IA are created for manipulating the formal causations. IA is one of the basic studies towards the next generation of intelligent computers known as cognitive computers. A wide range of applications of IA are identified and demonstrated in cognitive informatics and computational intelligence towards novel theories and technologies for machine-enabled inferences and reasoning.

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1. INTRODUCTION

Inference is a reasoning process that derives a causal conclusion from given premises. Formal inferences are usually symbolic and mathematical logic based, in which a causation is proven true by empirical observations, logical truths, mathematical equivalence, and/or statistical norms. Conventional logical inferences may be classified into the categories of deductive, inductive, abductive, and analogical inferences (Zadeh, 1965, 1975, 1999, 2004, 2008; Schoning, 1989; Sperschneider & Antoniou, 1991; Hurley, 1997; Tomassi, 1999; Wilson & Clark, 1988; Wang, 2007b, 2008a, 2011a; Wang et al., 2006), as well as *qualification* and *quantification* (Zadeh, 1999, 2004; Wang, 2007b, 2009c).

Studies on mechanisms and laws of inferences can be traced back to the very beginning of human civilization, which formed part of the foundations of various disciplines such as philosophy, logic, mathematics, cognitive science, artificial intelligence, computational intelligence, abstract intelligence, knowledge science, computational linguistics, and psychology (Zadeh, 1965, 1975, 2008; Mellor, 1995; Ross, 1995; Bender, 1996; Leahey, 1997; Wang, 2007c). Aristotle (1989) established *syllogism* that formalized inferences as logical arguments on propositions in which a conclusion is deductively inferred from two premises. Syllogism was treated as the fundamental methodology for inferences by Bertrand Russell in *The Principles of Mathematics* (Russell, 1903). Causality is a universal phenomenon of both the natural and abstract worlds because any rational state, event, action, or behavior has a cause. Further, any sequence of states, events, actions, or behaviors may be identified as a series of causal relations. In his classic work, *Principia: Mathematical Principles of Natural Philosophy* (Newton, 1687), Isaac Newton described a set of rules for inferences about nature known as the *experimental philosophy of causality* as follows:

- “**Rule 1:** We are to admit no more causes of natural things than such as are both true and sufficient to explain their appearances.”
- “**Rule 2:** Therefore to the same natural effects we must, as far as possible, assign the same causes.”
- “**Rule 3:** The qualities of bodies, which admit neither intension nor remission of degrees, and which are found to belong to all bodies within the reach of our experiments, are to be esteemed the universal qualities of all bodies whatsoever.”
- “**Rule 4:** In experimental philosophy we are to look upon propositions collected by general induction from phenomena as accurately or very nearly true, notwithstanding any contrary hypotheses that may be imagined, till such time as other phenomena occur, by which they may either be made more accurate, or liable to exceptions.”

In *A System of Logic*, John S. Mill identified six forms of *causal connections* between events in philosophy known as the methods of agreement, inverse agreement, double agreement, difference, residues, and concomitant variation (Mill, 1847). George Boole in *The Laws of Thought* studied the mathematical and logical laws of human thinking mechanisms and processes, where he perceived inference as operations of the mind based on logical and probability laws (Boole, 1854). Lotfi A. Zadeh created the fuzzy set theory and fuzzy logic since 1960s (Zadeh, 1965, 1975, 1999, 2004, 2008), which become one of the most applied theory for building fuzzy reasoning models in modern sciences and engineering. Fuzzy inferences are a powerful denotational mathematical means for rigorously dealing with degrees of matters, uncertainties, and vague semantics of linguistic variables, as well as for precisely reasoning the semantics of fuzzy causations.

Although there are various inference schemes and methods developed in a wide range of disciplines and applications, the framework of formal

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