Chapter 21
Equivalence between LDA/ QR and Direct LDA

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ABSTRACT

Singularity problems of scatter matrices in Linear Discriminant Analysis (LDA) are challenging and have obtained attention during the last decade. Linear Discriminant Analysis via QR decomposition (LDA/QR) and Direct Linear Discriminant analysis (DLDA) are two popular algorithms to solve the singularity problem. This paper establishes the equivalent relationship between LDA/QR and DLDA. They can be regarded as special cases of pseudo-inverse LDA. Similar to LDA/QR algorithm, DLDA can also be considered as a two-stage LDA method. Interestingly, the first stage of DLDA can act as a dimension reduction algorithm. The experiment compares LDA/QR and DLDA algorithms in terms of classification accuracy, computational complexity on several benchmark datasets and compares their first stages. The results confirm the established equivalent relationship and verify their capabilities in dimension reduction.

1. INTRODUCTION

Classical Linear Discriminant Analysis (LDA) algorithm usually suffers from the singularity problem (Ye et al., 2004; Chen et al., 2000) of scatter matrices. Linear Discrimiant Analysis via QR decomposition (LDA/QR) (Ye & Qi, 2000; Ye & Li, 2004) and Direct Linear Discriminant Analysis (DLDA) (Yu & Yang, 2005) are two LDA algorithms to solve this singularity problem. This paper proves the equivalence relationship between these two LDA algorithms.

LDA/QR is a LDA extension for coping with the singularity problem and was developed in (Ye & Qi, 2000). It improves the efficiency by introducing QR decomposition on a small-sized matrix, while achieving competitive discriminant performance. There are two stages-of-process in LDA/QR: 1) In the first stage, the separation between different classes is maximized via applying QR decomposition to a small-sized matrix. Remind that this stage can be used independently as a dimension reduction algorithm and it is called Pre-LDA/QR (Ye & Li, 2004); 2) The second stage refines the result by applying LDA to the so-called reduced scatter matrices (Ye & Qi, 2000) resulting from the first stage. The solution by LDA/QR has been proved to be equivalent to that by generalized LDA named pseudo-inverse LDA (Ye et al., 2004; Zhang et al., 2005).

DLDA (Yu & Yang, 2005) bases on the observation that the null space of between-class scatter matrix carries little useful discriminative information (Chen et al., 2000). Accordingly, it first diagonalizes between-class scatter matrix S_b and then adopts traditional eigen-analysis to discard the zero eigenvalues together with their corresponding eigenvectors. Then, it diagonalizes within-class scatter matrix S_w but keeps the zero eigenvalues since its null space carries the most useful discriminant information. Hence DLDA can make use of all discriminant information. One advantage of DLDA is that it can be applied to high-dimensional input space directly without any intermediate dimension reduction stage, such as PCA (Martinez & Kak 2001; Turk & Pentland, 1996; Yang & Yang, 2006), which has been used in the preprocessing stage of a well-known face recognition algorithm, namely fisherfaces (Belhumeur et al., 1997; Jing et al., 2006).

In this paper, we study on the equivalence between LDA/QR and DLDA. Ye and Li (2004) and Ye and Qi (2000)have demonstrated the equivalence between LDA/QR and pseudo-inverse LDA. Here, we also demonstrate the equivalence of DLDA to pseudo-inverse LDA. Then we can bridge LDA/QR and DLDA by using pseudo-inverse LDA. This achieves the equivalence between LDA/QR and DLDA. Meanwhile, we show that DLDA is a special case of pseudo-inverse LDA where the inverse of the between-class matrix is replaced by pseudo-inverse. Furthermore, we point out that DLDA can also be regarded as a two-stage LDA method such as LDA/QR, in which the first stage can be used independently as a dimension reduction algorithm. But LDA/QR is more efficient than DLDA, since singular value decomposition (SVD) (Golub & Loan, 2001) is used to the first stage of DLDA to maximize the between-class distance, whereas QR decomposition (Golub & Loan, 2001) is applied to LDA/QR, which is faster than SVD.

We have conducted a series of experiments to verify our theoretical proof. We make use of the first stage of the two LDA methods independently as dimension reduction algorithms to perform recognition on different datasets, i.e. human facial images and text. Our experimental results confirm our theoretical analysis.

There are three main contributions of this paper. First, we prove the equivalence between LDA/QR and DLDA. Second, we show that DLDA is a special case of pseudo-inverse LDA (where pseudo-inverse is applied to between-class scatter matrix). Finally, we demonstrate that DLDA can also be regarded as a two-stage LDA method, in which the first stage can be used independently as a dimension reduction algorithm.

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