

# Chapter 3

## Liveness, Deadlock– Freeness, and Siphons

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### ABSTRACT

*This chapter deals with the structure theory of Petri nets. The authors define the class of P/T systems, namely K-systems, for which the equivalence between controlled-siphon, deadlock-freeness, and liveness properties holds. Using the new structural notions of ordered transitions and root places, they revisit the non-liveness characterization of P/T systems satisfying the cs-property and define by syntactical manner new and more expressive subclasses of K-systems where the interplay between conflict and synchronization is relaxed.*

### 1. INTRODUCTION

Place/Transition (P/T) systems are a mathematical tool well suited for the modelling and analyzing systems exhibiting behaviour such as concurrency, conflict and causal dependency among events. The use of structural methods for the analysis of such systems presents two major advantages with respect to other approaches: the state explosion problem inherent to concurrent systems is avoided, and the investigation of the relationship between the behaviour and the structure (the graph theoretic and linear algebraic objects and properties associated with the net and initial marking) usually leads to a deep understanding of the system. Here we deal with liveness of a marking, i.e., the

fact that every transition can be enabled again and again. It is well known that this behavioural property is as important as formally hard to treat. Although some structural techniques can be applied to general nets, the most satisfactory results are obtained when the inter-play between conflicts and synchronization is limited. An important theoretical result is the controlled siphon property (Barkaoui & Peyre, 1996). Indeed this property is a condition which is necessary for liveness and sufficient for deadlock-freeness. The aim of this work is to define and recognize structurally a class of P/T systems, as large as possible, for which the equivalence between liveness and deadlock freeness holds. In order to reach such a goal, a deeper understanding of the causes of the

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non equivalence between liveness and deadlock-freeness is required.

This chapter is organized as follows. In section 2, we recall the basic concepts and notations of P/T systems. In section 3, we define a class of P/T systems, namely K-systems first introduced in (Barkaoui, Couvreur, & Klai, 2005), for which the equivalence between controlled-siphon property (cs-property), deadlock freeness, and liveness holds. In section 4, we revisit the structural conditions for the non liveness under the cs-property hypothesis. In section 5, we define by a syntactical manner several new subclasses of K-systems where the interplay between conflict and synchronization is relaxed. Such subclasses are characterized using the new structural notions of ordered transitions and root places. In section 6, we define two other subclasses of K-systems based on T-invariants. We conclude with a summary of our results and a discussion of an open question.

## 2. BASIC DEFINITIONS AND NOTATIONS

This section contains the basic definitions and notations of Petri nets' theory (Reisig, 1985) which will be needed in the rest of this chapter.

### 2.1. Place/Transition Nets

**Definition 1:** A P/T net is a weighted bipartite digraph  $N=(P, T, F, V)$  where:  $P \neq \emptyset$  is a finite set of node places;  $T \neq \emptyset$  is a finite set of node transitions;  $F \subseteq (P \times T) \cup (T \times P)$  is the flow relation;  $V: F \rightarrow \mathbb{IN}^+$  is the weight function (valuation).

**Definition 2:** Let  $N = (P, T, F, V)$  be a P/T net. The preset of a node  $x \in (P \cup T)$  is defined as  $\bullet x = \{ y \in (P \cup T) \text{ s.t. } (y, x) \in F \}$ , The postset of a node  $x \in (P \cup T)$  is defined as  $x^\bullet = \{ y \in (P \cup T) \text{ s.t. } (x, y) \in F \}$ , the preset (resp. postset) of a set of nodes is the union of the preset (resp. postset) of its elements. The

sub-net induced by a sub-set of places  $P' \subseteq P$  is the net  $N' = (P', T', F', V')$  defined as follows:  $T' = \bullet P' \cup P'^\bullet$ ;  $F' = F \cap ((P' \times T) \cup (T \times P'))$ ;  $V'$  is the restriction of  $V$  on  $F'$ . The sub-net induced by a sub-set of transitions  $T' \subseteq T$  is defined analogously.

**Definition 3:** Let  $N=(P, T, F, V)$  be a P/T net. A shared place  $p$  ( $|\bullet p| \geq 2$ ) is said to be homogenous iff:  $\forall t, t' \in p^\bullet, V(p, t) = V(p, t')$ . A place  $p \in P$  is said to be non-blocking iff:  $p^\bullet \neq \emptyset \Rightarrow M \text{ in } \bullet p \in \bullet p \{ V(p, t) \} \geq M \text{ in } \bullet p \{ V(p, t) \}$ . If all shared places of  $P$  are homogenous, then the valuation  $V$  is said to be homogenous. The valuation  $V$  of a P/T net  $N$  can be extended to the application  $W$  from  $(P \times T) \cup (T \times P) \rightarrow \mathbb{IN}$  defined by:  $\forall u \in (P \times T) \cup (T \times P), W(u) = V(u)$  if  $u \in F$  and  $W(u) = 0$  otherwise.

**Definition 4:** The matrix  $C$  indexed by  $P \times T$  and defined by  $C(p, t) = W(t, p) - W(p, t)$  is called the incidence matrix of the net. An integer vector  $f \neq 0$  indexed by  $P$  ( $f \in \mathbb{Z}^P$ ) is a P-invariant iff  $f^\bullet \cdot C = 0$ . An integer vector  $g \neq 0$  indexed by  $T$  ( $g \in \mathbb{Z}^T$ ) is a T-invariant iff  $C \cdot g = 0$ .  $\|f\| = \{ p \in P | f(p) = 0 \}$  (resp.  $\|g\| = \{ t \in T | g(t) = 0 \}$ ) is called the support of  $f$  (resp. of  $g$ ). We denote by  $\|f\|^+ = \{ p \in P | f(p) > 0 \}$  and by  $\|f\|^- = \{ p \in P | f(p) < 0 \}$ .  $N$  is said to be conservative iff there exists a P-invariant  $f$  such that  $\|f\| = \|f\|^+ = P$ .

### 2.2. Place/Transition Systems

**Definition 5:** A marking  $M$  of a P/T net  $N = (P, T, F, V)$  is a mapping  $M: P \rightarrow \mathbb{IN}$  where  $M(p)$  denotes the number of tokens contained in place  $p$ . The pair  $(N, M_0)$  is called a P/T system with  $M_0$  as initial marking. A transition  $t \in T$  is said to be enabled under  $M$ , in symbols  $M \xrightarrow{t}$ , iff  $\forall p \in \bullet t: M(p) \geq V(p, t)$ . If  $M \xrightarrow{t}$ , the transition  $t$  may occur, resulting in a new marking  $M'$ , in symbols  $M \xrightarrow{t} M'$ , with:  $M'(p) = M(p) - W(p, t) + W(t, p), \forall p \in P$ . The set of all reachable markings, in symbols  $R(M_0)$ , is the smallest

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