Hyperbolic Space for Interactive Visualization

Jörg Andreas Walter
University of Bielefeld, Germany

INTRODUCTION

For many tasks of exploratory data analysis, visualization plays an important role. It is a key for efficient integration of human expertise — not only to include his background knowledge, intuition and creativity, but also his powerful pattern recognition and processing capabilities. The design goals for an optimal user interaction strongly depend on the given visualization task, but they certainly include an easy and intuitive navigation with strong support for the user's orientation.

Since most of available data display devices are two-dimensional — paper and screens — the following problem must be solved: finding a meaningful spatial mapping of data onto the display area. One limiting factor is the "restricted neighborhood" around a point in a Euclidean 2-D surface. *Hyperbolic spaces* open an interesting loophole. The extraordinary property of exponential growth of neighborhood with increasing radius — around all points — enables us to build novel displays and browsers. We describe a versatile hyperbolic data viewer for building data landscapes in a non-Euclidean space, which is intuitively browseable with a very pleasing focus and context technique.

BACKGROUND

The Hyperbolic Space H2

2,300 years ago, the Greek mathematician Euclid founded his geometry on five axioms. The fifth, the "parallel axiom," appeared unnecessary to many of his colleagues. And they tried hard to prove it derivable — without success. After almost 2,000 years Lobachevsky (1793-1856), Bolyai (1802-1860), and Gauss (1777-1855) negated the axiom and independently discovered the non-Euclidean geometry. There exist only two geometries with constant non-zero curvature. Through our sight of common spherical surfaces (e.g., earth, orange) we are familiar with the *spherical geometry* and its constant *positive* curvature. Its counterpart with constant *negative* curvature is known as the hyperbolic plane H2 (with analogous generalizations to higher dimensions). Unfortunately, there is no "good" embedding of the H2 in R^3, which makes it harder to grasp the unusual properties of the H2. Local patches resemble the situation at a saddle point, where the neighborhood grows faster than in flat space. Standard textbooks on Riemannian geometry (see, e.g., Morgan, 1993) examine the relationship and expose that the area a of a circle of radius r are given by

$$a(r) = 4\pi \sinh^2(r/2) \tag{1}$$

This bears two remarkable asymptotic properties: *(i)* for small radius r the space "looks flat" since $a(r) = \pi r^2$. *(ii)* For larger r both grow *exponentially* with the radius. As observed in Lamping & Rao (1994, 1999) and Lamping et al. (1995), this property makes the hyperbolic space ideal for embedding hierarchical structures (since the number of leaves grows exponentially with the tree depth). This led to the development of the *hyperbolic tree browser* at Xerox Parc (today *starviewer:* see *www.inxight.com*).

The question how effective is visualization and navigation in the hyperbolic space was studied by Pirolli et al. (2001). By conducting eye-tracker experiments they found that the focus+context navigation can significantly accelerate the "information foraging." Risden et al. (2000) compared traditional and hyperbolic browsers and found significant improvement in task execution time for this novel display type.

MAIN THRUST

In order to use the visualization potential of the H2 we must solve two problems:

- *(P1)* How can we "accommodate" the data in the hyperbolic space, and
- *(P2)* how to project the H2 onto a suitable display?

In the following the answers are described in reverse sequence, first for the second problem (P2) and then three principal techniques for P1, which are today available for different data types.

Figure 1. Regular H2 tessellation with equilateral triangles (here 8 triangles meet at each vertex). Three snapshots of a simple focus transfer are visible. Note the circular appearance of lines in the Poincaré disk PD and the fish-eye-lens effect: the triangles in the center appear larger and take less space in regions further away.
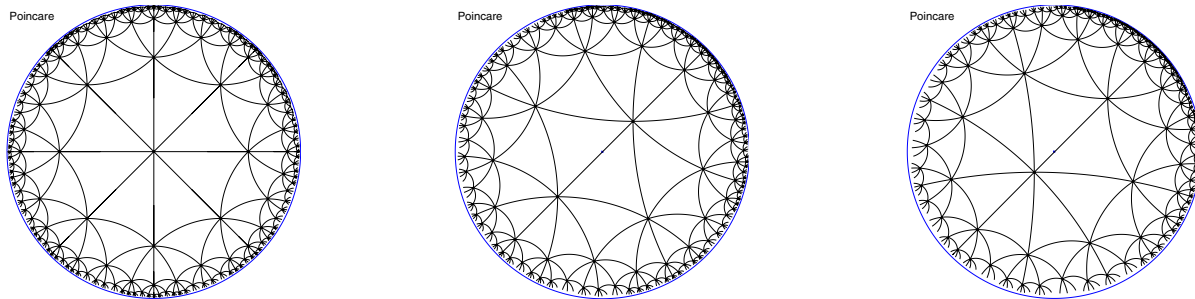
Solution for P2: Poincaré Disk PD

For practical and technological reasons most available displays are flat. The perfect projection into the flat display area should preserve length, area, and angles (=form). But it lays in the nature of a curved space to resist the attempt to simultaneously achieve these goals. Consequently several projections or *maps* of the hyperbolic space were developed, four are especially well examined: (i) the Minkowski, (ii) the upper-half plane, (iii) the Klein-Beltrami, and (iv) the Poincaré or disk mapping. For our purpose the latter is particularly suitable. Its main characteristics are:

- **Display Compatibility:** The infinite large area of the H2 is mapped entirely into a circle, the Poincaré disk PD.
- **Circle Rim is Infinity ∞:** All remote points are close to the rim, without touching it.
- **Focus+Context:** The *focus* can be moved to each location in H2, like a "fovea." The zooming factor is 0.5 in the center and falls (exponentially) off with distance to the fovea. Therefore, the context appears very natural. As more remote things are, the less spatial representation is assigned in the current display (compare *Figure 1*).
- **Lines Become Circles:** All H2-lines appear as circle arc segments of centered straight lines in PD (both belong to the set of so-called "generalized circles"). There extensions cross the PD-rim always perpendicular on both ends.
- **Conformal Mapping:** Angles (and therefore form) relations are preserved in PD, area and length relations obviously not.

- **Regular Tessellations** with triangles offer richer possibilities than the R^2. It turns out that there is an infinite set of choices to tessellate H2: for any integer $n \geq 7$, one can construct a regular tessellations in which n triangles meet at each vertex (in contrast to the plane with allows only $n=3,4,6$ and the sphere only $n=3,4,5$). *Figure 1* depicts an examples for $n=8$.
- **Moving Around and Changing the Focus:** For changing the focus point in PD we need a translation operation, which can be bound to mouse click and drag events. In the Poincaré disk model the Möbius transformation $T(z)$ is the appropriate solution. By describing the Poincaré disk PD as the unit circle in the complex plane, the isometric transformations for a point $z \subset PD$ can be written as

$$z' = T(z; c, \theta) = (\theta z + c)/(c*\theta z + 1), \text{ with } |\theta| = 1, |c| < 1. \tag{2}$$

Here the complex number θ describes a pure rotation of PD around the origin 0 (the star * denotes complex conjugation). The following translation by c maps the origin to c and $-c$ becomes the new center 0 (if $\theta=1$). The Möbius transformations are also called the "circle automorphies" of the complex plane, since they describe the transformations from circles to (generalized) circles. Here they serve to translate H2 straight lines to lines — both appearing as generalized circles in the PD projection. For further details, see for example, Lamping & Rao (1999) or Walter (2004).

Three Layout Techniques in H2

Now we turn to the question raised earlier: how to accommodate data in the hyperbolic space. In the following

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