

# Multiple-Objective Fractional TP with Impurities

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## INTRODUCTION

The main objectives of this chapter are to present

1. Literature review of fractional transportation problems for business optimisation,
2. Multiple-objective fractional transportation problems with impurities,
3. Summarized solution procedure for solving multiple-objective fractional transportation problem with impurities and
4. Point out future research directions in this field.

In various business applications of optimization a function, characterized by one or several ratios like profit/capital, total tax/total public expenditure, may be maximized or minimized. Such optimization problems are commonly called Fractional Programming or Fractional Programs.

Mathematically, fractional program is of the form:

$$P_1 \max(\min) q(x) = \frac{f(x)}{g(x)} \quad (1)$$

subject to

$$h_i(x) \leq b_i, \quad i = 1, 2, \dots, m \quad (2)$$

$$x \geq 0 \quad (3)$$

Here  $x \in \mathbb{R}^n$  is a  $(n \times 1)$  vector which is to be determined,  $f(x)$ ,  $g(x)$ , and  $h_i(x)$  are real valued scalar functions defined on a subset of  $\mathbb{R}^n$ . Assume  $g(x) > 0$  for all  $x$ . For negative  $g(x)$ ,

$$q(x) = \frac{-f(x)}{-g(x)} \quad (4)$$

may be used instead.

Fractional programs arise in various businesses as well as in other areas like:

1. Maximizing relative usage of raw material-stock cutting problem
2. Maximizing return/risk
3. Maximizing return/cost
4. Minimizing expected cost/time

These ratios arise in portfolio selection, resource allocation, the analysis of financial enterprises and undertaking, finance, maintenance, Markov renewal programs, transportation, etc.

Transportation problems with fractional objective functions arise in many real life situations where an individual, or a group, or a commodity is faced with the problem of maintaining good ratios between some very important crucial parameters concerned with the transportation of commodities from certain sources to various destinations. This may be the situation for an enterprise board confronted with optimization of total actual/total standard transportation cost or total return/total investment on machines when acquired from fac-

ories  $i$  to workshops  $j$ . A fractional transportation problem can be formulated as (Swarup, 1966):

$$P_2 \min z = \frac{\sum_i \sum_j c_{ij} x_{ij}}{\sum_i \sum_j d_{ij} x_{ij}} \quad (5)$$

subject to

$$\sum_{j=1}^N x_{ij} = a_i \quad (i=1, \dots, M) \quad (6)$$

$$\sum_{i=1}^M x_{ij} = b_j \quad (j=1, \dots, N) \quad (7)$$

$$x_{ij} \geq 0, \text{ for all } i \text{ and } j \quad (8)$$

Here  $c_{ij}$  and  $d_{ij}$  represent the proportional contribution to the value of the actual and standard cost objective function of shipping one unit of commodity from origin  $i$  to destination  $j$ .  $a_i$  is the amount of commodity available at origin  $i$ ,  $b_j$  is the requirement of the commodity at destination  $j$ ,  $x_{ij}$  is the amount of commodity transported from origin  $i$  to destination  $j$ .

## BACKGROUND

Fractional Transportation models have been applied to a broad class of decision problems. Swarup (1966) studied a transportation technique for linear fractional functional programming problem and derived conditions for optimality for the solution of such problems in terms of simplex multipliers under the assumption that the denominator is always positive. Kanchan et al. (1981) investigated transportation techniques in linear plus linear fractional programming of the form:

$$\min f(x) = \sum_i \sum_j c_{ij} x_{ij} + \frac{\sum_i \sum_j d_{ij} x_{ij}}{\sum_i \sum_j q_{ij} x_{ij}} \quad (9)$$

subject to (6), (7), (8).

The objective function (9) appears for a whole class of economic problems, for instance, optimizing the remuneration fund and the profitability of an economic enterprise.

Aggarwal (1972) introduced transportation technique for quadratic fractional programming problems of the form:

$$\min z = \frac{\left[ \sum_i \sum_j c_{ij} x_{ij} \right]^2}{\left[ \sum_i \sum_j d_{ij} x_{ij} \right]^2} \quad (10)$$

subject to (6), (7), (8).

Guzel et al. (2012) transformed a fractional transportation problem with interval coefficient to a classical transportation problem by expanding the order 1<sup>st</sup> Taylor polynomial series with multi variables. Gupta and Arora (2012) discussed a paradox in a capacitated transportation problem where the objective function is a ratio of two linear functions consisting of variable costs and profits respectively. Acharya et al. (2013) developed a new algorithm for obtaining the optimum solution of Discounted Generalized Transportation Problem. Joshi and Gupta (2011) investigated the transportation problem with fractional objective function when the demand and supply quantities are varying. A set of mathematical programs was obtained to determine the objective value. Prakash et al. (2008) considered a cost–time trade-off bulk transportation problem with the objectives to minimize the total cost and duration of bulk transportation without according priorities to them. Edokpia and Ohikhuare (2011) utilized a linear programming technique in solving the transportation problem of a beverage producing company in Nigeria with a view to minimizing the total transportation cost and obtaining an optimal schedule bearing in mind the present transportation policy of the company. Khurana and Arora (2006) considered a transportation problem with an objective function as the sum of a linear and

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