Simulation Optimization and a Case Study

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INTRODUCTION

This chapter is a continuation of our chapters "Simulation Optimization Via Metamodeling Approach" and "Simulation Optimization For Finite Parameter Space" found elsewhere in this volume. In this chapter, we investigate gradient-based simulation optimization (GBSO) methods, including Perturbation Analysis (PA), Likelihood Ratio (LR), Finite Difference (FD), and Frequency Domain Analysis (FDA). We also provide discussion on several commercial software packages from the literature developed for simulation optimization (SO). Last, we present an OptQuest application for a real case study.

BACKGROUND

There are a large number of surveys on GBSO techniques. Andradottir (1998a, b) presents a review of SO techniques by focusing on both GBSO methods and random search methods. Fu (1994a, b; 2002) presents a comprehensive overview for SO as well as GBSO techniques. Kim (2006) also reviews methods for SO by focusing on GBSO methods. Barton and Meckesheimer (2006) provide a classification for SO techniques. According to their survey SO problems fall into two categories: direct gradient and metamodel methods. Direct gradient methods estimate the gradient of the simulation response, and then resort to stochastic gradient-based techniques such as stochastic approximation. Ammeri, Chabchoub, Hachicha, & Masmpudi (2010) present a survey study on methods for SO according to the characteristics of problems such as shape of the response surface and parameter spaces (discrete or continuous). There are also case study implementations for SO (Gupta, Evans, & Heragu, 2013; Melouk, Freeman, Miller, & Dunning, 2013; Ozdemir, Yucesan, & Herer, 2013).

Gradient-Based Simulation Optimization AND Commercial Software Packages

Issues, Controversies, Problems

Differentiation of a function is often used to find an optimum point for that function. Although a gradient-based approach requires a mathematical expression of the objective function, when such mathematical expression cannot be obtained, there is a need to use an estimation technique to initiate the solution procedure. PA, LR, FD, and FDA methods are the four successfully used gradient estimators that we discuss in this chapter. We also discuss several simulation commercial software packages with associated optimization tools.

DOI: 10.4018/978-1-4666-5202-6.ch194

Perturbation Analysis

PA is a sample path technique for analyzing changes in the output of stochastic systems based on the system's input variable changes. In this approach, the aim is to estimate sensitivities of the outputs based on the input variables with no additional runs. PA is mainly used to estimate the gradient of the output during the simulation. Later, this estimated gradient is used for optimization. Infinitesimal perturbation analysis (IPA) is one of the widely known methods among the gradient estimator techniques. IPA is straightforward to implement as well as computationally efficient. However, it cannot be implemented on all type of problems (Ho, & Cao, 1991; Glasserman 1991; Cao 1994).

In IPA the main idea is that in a system, if an input variable is perturbed by an infinitesimal amount, the sensitivity of the output to the parameter can be estimated by tracing its pattern of propagation. IPA assumes that an infinitesimal perturbation in an input variable does not affect the sequence of events but only makes their occurrence times slide smoothly. The fact that all derivatives can be derived from a single simulation run represents a significant advantage in terms of computational efficiency. On the other hand, the estimators derived using IPA are often biased and inconsistent. According to Glynn (1989b), when both IPA and LR methods are applied to a given problem, the IPA gradient estimator is more efficient. Smoothed perturbation analysis (SPA) is a very general and well-developed extension of IPA (Gong, & Ho, 1987). Although its implementation is wide, it is very problem dependent (Fu, & Hu, 1997). There are also some extensions of PA techniques such as rare perturbation analysis (Bremaud, & Vazquez-Abad, 1992), structural IPA (Dai & Ho, 1995), discontinuous perturbation analysis (Shi, 1996) and augmented IPA (Gaivoronski, Shi, & Sreenivas, 1992). Finite perturbation analysis is developed to estimate the gradient of an output due to the effect of a finite perturbation (Ho, Cao, & Cassandras, 1983). For further references see the books by Ho and Cao (1991) and Glasserman (1991).

Likelihood Ratio

The LR method is also called the score function method. In this method, the basic idea is to differentiate the underlying probability measure of the system. However, it can also be viewed as a special case of importance sampling. An overview of LR estimators and their potential use in simulation optimization can be found in Glynn (1987). In that study, two algorithms allowing us to estimate the gradient of a simulation output with respect to its parameters are discussed. A variation of LR is provided by Rubenstein (1989). His approach can be used in estimation of Hessians and higher level gradients to be incorporated in Newton's method. After estimating the gradient, one can implement an available search technique for the optimization search.

The LR method can be applied on systems where IPA fails. However, this time the estimator may have large variance. The LR method is suitable for transient and regenerative SO problems. For a regenerative process, the steady state value of an output can be expressed as a ratio of two expected values. In the construction of an LR it is important to obtain desirable computational and variability characteristics (Glynn, 1989b). Additional details can be found in Glynn (1989a), and Reiman and Weiss (1986).

Finite Differences

FD is the crudest method of estimating the gradient (Azadivar, 1992). Partial derivatives of $f(X)$ in this case are estimated by (1):

$$\delta f \big/ \delta X_i = \left[\left(f\left(X_1, \ldots, X_i + \Delta X_i, \ldots, X_p\right) - f\left(X_1, \ldots, X_p\right) \right] \big/ \Delta X_i \qquad (1)$$

In this technique, at least $n+1$ evaluations of the simulation model will be required to estimate