Stochastic Programming and Value Based Decisions

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INTRODUCTION

Value based decision making is an integrated approach used to guide actions and resolve problems in complex situations where human activities are vital to achieving the best result. The objective of value based decision making is to develop a mathematical framework (econometric) for management and modeling of complex systems. The aspiration for measurements, quantity estimations and prognosis is natural but the correct assessment requires careful analysis of the terms measurement, formalization and admissible mathematical operations. Often, in complex processes, there is a lack of measurements or even clearly identifiable scales for the basic heuristic information. The basic common source of information here are the human preferences. According to social-cognitive theories, people's strategies are guided both by internal expectations about their own capabilities of getting results, and by external feedback (Bandura, 1986). Internal human expectations and assessments are generally expressed by qualitative preferences. Probability theory and expected utility theory address decision making under these conditions (Fishburn, 1970; Keeney & Raiffa, 1993).

The American psychologists Griffiths and Tenenbaum by analyzing intuitive evaluations in the conditions of repetitive life situations have proved the statistical optimality of human assessment (Griffiths & Tenenbaum, 2006). The major idea of this study is that the new data is interpreted in the framework of a built in consciousness probability model. This means that the Bayesian approach is a natural basis on which human beings have formed their decisions (using their previous empirical experience). In such case the utility theory and its prescription to make decisions based on value (utility) model as mathematical representation of the preferences has another scientific validation of the axiomatic approach in the decision making. This modeling and its implementation in design is one of the directions of the value driven design (Collopy & Hollingsworth, 2009). Value-driven design is a systems engineering strategy which enables multidisciplinary design optimization by providing designers with an objective function.

People's preferences contain uncertainty of probabilistic nature due to the qualitative type of both the empirical expert information and human notions. A possible approach for solution of these problems is the stochastic programming (Aizerman, Braverman, & Rozonoer, 1970).

The uncertainty of the subjective preferences could be considered as an additive noise that could be eliminated, as is typical in the stochastic approximation procedures and machine-learning based on the stochastic programming.

The main objective of the chapter is the productive merger of the mathematical exactness with the empirical uncertainty in the human notions. The focus is inclusion of the decision maker mathematically in the modeling and in the main objective function. The approach described in the chapter permits representation of the individual's

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preferences as value/utility function, evaluated as stochastic programming machine learning for value based decision making which supports value driven design.

BACKGROUND

The description of the value based decision requires basic analytical representation of the DM's preferences. The mathematical description on such a fundamental level requires basic mathematical terms like sets, relations and operations over them, and their gradual elaboration to more complex and specific terms like functions, operators on mathematically structured sets as well, and equivalency of these descriptions with respect to a given real object. In the last aspect of equivalency of the mathematical descriptions we enter the theory of measurements and scaling (Luce, Krantz, Suppes, & Tversky, 1990; Pfanzagl, 1971).

We start by defining the term System with relations. *System with relations* (SR) is called the set $\boldsymbol{A}$ in conjunction with a set of relations $\boldsymbol{R_i}$, $\boldsymbol{i}\in\boldsymbol{I}$, $\boldsymbol{I}=\{1,2,3,...,n\}$ defined over it and we denote it by $(\boldsymbol{A}, (\boldsymbol{R_i}), \boldsymbol{i}\in\boldsymbol{I})$. In this manner we introduce structure in the set $\boldsymbol{A}$. Relation of *congruency* is called a relation of equivalency ($\approx$) (reflexive, symmetric and transitive) defined over the basic set $\boldsymbol{A}$, if the property of *substitution* is satisfied, i.e. from the fulfillment of relations $(x_1, x_2, x_3, ..., x_{hi})\in\boldsymbol{A}^{hi}$ and $(x_j\approx y_j)$ for every j=1, 2, 3 4,...,h_i it follows that $\boldsymbol{R_i}(x_1, x_2, x_3, ..., x_{hi})=\boldsymbol{R_i}(y_1, y_2, y_3, ..., y_{hi})$ for $\forall$ $\boldsymbol{i}$, $\boldsymbol{i}\in\boldsymbol{I}$. We say that the relation of equivalency $(\approx_2)$ is coarser than the equivalency $(\approx_1)$, if the inclusion $(\approx_1) \subseteq (\approx_2)$ is satisfied. It is known that there exists a coarsest relation $(\approx_A)$ over SR $(\boldsymbol{A}, (\boldsymbol{R_i}), \boldsymbol{i}\in\boldsymbol{I})$. This means that if two elements are in congruency relation $(x\approx_A y)$, then they are undistinguishable, with respect to the properties, in the set $\boldsymbol{A}$, described with the set of relations $((\boldsymbol{R_i}), \boldsymbol{i}\in\boldsymbol{I})$. If we factorize the set $\boldsymbol{A}$ by the coarsest congruency $(\approx_A)$, then in the factor set $\boldsymbol{A}/\approx_A$ the congruency $(\approx_A)$ is in fact equality (=). A SR $(\boldsymbol{A}, (\boldsymbol{R_i}), \boldsymbol{i}\in\boldsymbol{I})$, in which the congruency $(\approx_A)$ is coarsest is called *irreducible*. In this case SR $(\boldsymbol{A}/\approx_A, (\boldsymbol{R_i}), \boldsymbol{i}\in\boldsymbol{I})$ is irreducible.

A homomorphism is an image f, f: $\boldsymbol{A}\rightarrow\boldsymbol{B}$ between two SR $(\boldsymbol{A}, (\boldsymbol{R_i}), \boldsymbol{i}\in\boldsymbol{I})$ and $(\boldsymbol{B}, (\boldsymbol{S_i}), \boldsymbol{i}\in\boldsymbol{I})$ from the same type, for which $\forall$ $\boldsymbol{i}$, $\boldsymbol{i}\in\boldsymbol{I}$ and $(x_1, x_2, x_3, ..., x_{hi})\in \boldsymbol{R_i}$ is satisfied $\boldsymbol{R_i}(x_1, x_2, x_3, ..., x_{hi}) \Leftrightarrow \boldsymbol{S_i}(f(x_1), f(x_2), f(x_3), ..., f(x_{hi}))$.

Definition: We call k-dimensional scale every homomorphism from irreducible empirical system into the number system SR $(\boldsymbol{A}, (\boldsymbol{Q_i}), \boldsymbol{i}\in\boldsymbol{I})$.

The empirical system of relations SR is an object from the reality with the properties described by the relations in SR, while the numbered system of relations SR is a mathematical object which reflects the interesting to us properties in the real object.

In the definition for the scale the correspondence f_{0}: $\boldsymbol{A}\rightarrow \boldsymbol{R}^{\kappa}$ is not simply defined ($\boldsymbol{R}^{\kappa}$ is the k-ary Cartesian product of the set of the real numbers). In general sense, there exists entire class of scales converting the irreducible empirical system of relations SR $(\boldsymbol{A}, (\boldsymbol{R_i}), \boldsymbol{i}\in\boldsymbol{I})$ into the number system SR $(\boldsymbol{R}^{\kappa}, (\boldsymbol{S_i}), \boldsymbol{i}\in\boldsymbol{I})$. We denote this class of homomorphisms by $\aleph(\boldsymbol{A}, \boldsymbol{R}^{\kappa})$ (injective in its essence because the empirical system is irreducible and surjective with regard to $f(\boldsymbol{A})$). Let $\boldsymbol{A_0}$ be a subset of $\boldsymbol{A}$. We denote by $G_{\boldsymbol{A}}(A_0)$ all injective homomorphisms, inclusion (*partial endomorphism*) from SR $(\boldsymbol{A_0}, (\boldsymbol{R_i}), \boldsymbol{i}\in\boldsymbol{I})$ in SR $(\boldsymbol{A}, (\boldsymbol{R_i}), \boldsymbol{i}\in\boldsymbol{I})$. If a scale $f_0\in\aleph(\boldsymbol{A}, \boldsymbol{R}^{\kappa})$ is given, then we can characterize the whole class of scales $\aleph(\boldsymbol{A}, \boldsymbol{R}^{\kappa})$ in the following way: $\aleph(\boldsymbol{A}, \boldsymbol{R}^{\kappa}) = \{\gamma_{\circ} f_0 /$ where $\gamma\in G_{\boldsymbol{R}}{}^{\kappa}(f_0(\boldsymbol{A}))\}$. Or in other words two scales are equivalent with precision to partial endomorphism $\gamma\in G_{\boldsymbol{R}}{}^{\kappa}(f_0(\boldsymbol{A}))$. The elements of $G_{\boldsymbol{R}}{}^{\kappa}(f_0(\boldsymbol{A}))$ are called *admissible manipulations* of the scale f_0 (Pfanzagl, 1971).

An example is the measurement of the temperature. The scale $f_0(.)$ is the temperature in Celsius. Then every partial endomorphism is an affine correspondence of the type $\gamma(x)=\boldsymbol{a}x+\boldsymbol{b}$, $\boldsymbol{a}\in\boldsymbol{R}$, $\boldsymbol{b}\in\boldsymbol{R}$ and $\boldsymbol{a}>\boldsymbol{0}$.

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