

# Guide Manifold Alignment by Relative Comparisons

**Liang Xiong**

*Tsinghua University, China*

**Fei Wang**

*Tsinghua University, China*

**Changshui Zhang**

*Tsinghua University, China*

## INTRODUCTION

When we are faced with data, one common task is to learn the correspondence relationship between different data sets. More concretely, by learning data correspondence, samples that share similar intrinsic parameters, which are often hard to estimate directly, can be discovered. For example, given some face image data, an alignment algorithm is able to find images of two different persons with similar poses or expressions. We call this technique the alignment of data. Besides its usage in data analysis and visualization, this problem also has wide potential applications in various fields. For instance, in *facial expression recognition*, one may have a set of standard labeled images with known expressions, such as *happiness*, *sadness*, *surprise*, *anger* and *fear*, of a particular person. Then we can recognize the expressions of another person just by aligning his/her facial images to the standard image set. Its application can also be found directly in pose estimation. One can refer to (Ham, Lee & Saul, 2005) for more details.

Although intuitive, without any premise this alignment problem can be very difficult. Usually, the samples are distributed in high-dimensional observation spaces, and the relation between features and samples' intrinsic parameters can be too complex to be modeled explicitly. Therefore, some hypotheses about the data distribution are made. In the recent years, the manifold assumption of data distribution has been very popular in the field of data mining and machine learning. Researchers have realized that in many applications the samples of interest are actually confined to particular subspaces embedded in the high-dimensional feature space (Seung & Lee, 2000; Roweis & Saul, 2000). Intuitively, the manifold

assumption means that certain groups of samples are lying in a non-linear low-dimensional subspace embedded in the observation space. This assumption has been verified to play an important role in human perception (Seung & Lee, 2000), and many effective algorithms are developed under it in the recent years. Under the manifold assumption, structural information of data can be utilized to facilitate the alignment. Figure 1 illustrates two 1-D manifolds embedded in a 2-D plane.

Besides, since we do not know the relationship between observations and the underlying parameters, additional supervisions are needed to guide the aligning process. Usually, we will require information about a subset of samples and the structure of the whole data set in order to infer about the rest of samples. Thus,

*Figure 1. Two 1-D manifolds embedded in a 2-D plane*

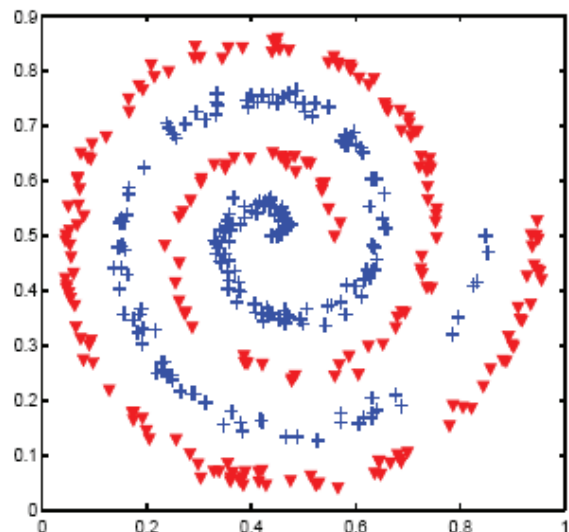
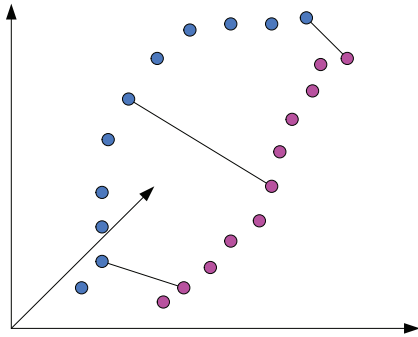


Figure 2. Guide the alignment of two manifolds by pairwise correspondence. A black line represents the supervision which indicates two samples with the same underlying parameters.



the alignment is often done in a semi-supervised way. The most common supervision used is pairwise correspondence, which specifies two samples that share the same parameters as shown in figure 2.

Take the facial expression alignment for example. In this task we are given face images of two different persons A and B, and the problem is to find two images, one of which is from A and the other is from B, with similar facial expression. Template matching in the feature space is not feasible here since different faces may have very different appearances. At present, directly estimating the expression parameters is also difficult because of the limitation of our knowledge and the variability of data. So we assume that images from the same person are lying on a low-dimensional manifold. Now the problem seems easier because we are now dealing with two structures, such as two curves, instead of discrete points. However, we still do not know how these two structures should be aligned. Then supervisions are needed to tell us how they correspond.

There have already been several methods proposed to align manifolds in a semi-supervised way (Ham, Ahn & Lee, 2006; Ham, Lee & Saul, 2003; Ham, Lee & Saul, 2005; Verbeek, Roweis & Vlassis, 2004; Verbeek & Vlassis, 2006). Specifically, they usually assumed that some pair-wise correspondences of samples in different data sets were already known, and then this information would be used to guide the alignment. However, in practice it might be difficult to obtain and use such information since:

1. The sizes of data sets can be very large, then finding high-quality correspondences between them can be very time consuming and even intractable.
2. There may be ambiguities in the images, which makes explicit matching a hard task. Brutally determine and enforce these unreliable constraints may lead to poor results;
3. Sometimes it may be hard to find the exact correspondences when the available samples are scarce. This situation may happen when the data source is restricted and users are only allowed to access a small subset.

To solve the above problems, we could apply another type of supervision to guide the process of manifold alignment. In particular, we consider a relative and qualitative supervision of the form “A is closer to B than A is to C”. We believe that this type of information is more easily available in practice than traditional correspondence-based information. With the help of such information, we show that the manifold alignment problem can be formulated as a *Quadratically Constrained Quadratic Programming* (QCQP) (Boyd & Vandenberghe, 2004) problem. To make the optimization tractable, we further relax it to a *Semi-Definite Programming* (SDP) (Boyd & Vandenberghe, 2004) problem, which can be readily solved by popular optimization software packages. Besides, under this formulation we are able to incorporate both relative relations and correspondences to align manifolds in a very flexible way.

## BACKGROUND

Before alignment, we need an infrastructure to capture the structure of each manifold. First, we will introduce manifold embedding.

The embedding algorithms find coordinates in a novel low-dimensional space for the data points so that certain geometric relations between them in the feature space are retained in the embedded space. Here we complete this task by graph regularization as in *Laplacian Eigenmaps* (Belkin & Niyogi, 2003). An undirected weighted graph is constructed over the data points to characterize their relations. Then this method aims at preserving the connectivity in this graph *i.e.* if two points are originally connected, then after embedding they are also connected; if two points

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