

Transferable Belief Model

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INTRODUCTION

This note is a very short presentation of the transferable belief model (TBM), a model for the representation of quantified beliefs based on belief functions. Details must be found in the recent literature.

The TBM covers the same domain as the subjective probabilities except probability functions are replaced by belief functions which are much more general. The model is much more flexible than the Bayesian one and allows the representation of states of beliefs not adequately represented with probability functions. The theory of belief functions is often called the Dempster-Shafer's theory, but this term is unfortunately confusing.

The Various Dempster-Shafer's Theories

Dempster-Shafer's theory covers several models that use belief functions. Usually their aim is in the modeling of someone's degrees of belief, where a degree of belief is understood as strength of opinion. They do not cover the problems of vagueness and ambiguity for which fuzzy sets theory and possibility theory are more appropriate.

Beliefs result from uncertainty. Uncertainty can result from a random process (the objective probability case), or from a lack of information (the subjective case). These two forms of uncertainty are usually quantified by probability functions.

Dempster-Shafer's theory is an ambiguous term as it covers several models. One of them, the "transferable belief model" is a model for the representation of quantified beliefs developed independently of any underlying probability model. Based on Shafer's initial work (Shafer, 1976) it has been largely extended since (Smets, 1998; Smets & Kennes, 1994; Smets & Kruse, 1997).

The Representation of Quantified Beliefs

Suppose a finite set of worlds Ω called the frame of discernment. The term "world" covers concepts like state of affairs, state of nature, situation, context, value

of a variable... One world corresponds to the actual world. An agent, denoted You (but it might be a sensor, a robot, a piece of software), does not know which world corresponds to the actual world because the available data are imperfect. Nevertheless, You have some idea, some opinion, about which world might be the actual one. So for every subset A of Ω , You can express Your beliefs, i.e., the strength of Your opinion that the actual world belongs to A . This strength is denoted $\text{bel}(A)$. The larger $\text{bel}(A)$, the stronger You believe that the actual world belongs to A .

Credal vs. Pignistic Levels

Intrinsically beliefs are not directly observable properties. Once a decision must be made, their impact can be observed.

In the TBM, we have described a two level mental model in order to distinguish between two aspects of beliefs, belief as weighted opinions, and belief for decision making (Smets, 2002a). The two levels are: the credal level, where beliefs are held, and the pignistic level, where beliefs are used to make decisions (credal and pignistic derive from the Latin words "credo", I believe and "pignus", a wage, a bet).

Usually these two levels are not distinguished and probability functions are used to quantify beliefs at both levels. Once these two levels are distinguished, as done in the TBM, the classical arguments used to justify the use of probability functions do not apply anymore at the credal level, where beliefs will be represented by belief functions. At the pignistic level, the probability function needed to compute expected utilities are called pignistic probabilities to enhance they do not represent beliefs, but are just induced by them.

BACKGROUND

Belief Function Inequalities

The TBM is a model developed to represent quantified beliefs. The TBM departs from the Bayesian approach

in that we do not assume that bel satisfies the additivity encountered in probability theory. We get inequalities like : $\text{bel}(A \cup B) \geq \text{bel}(A) + \text{bel}(B) - \text{bel}(A \cap B)$.

Basic Belief Assignment

Definition 2.2

Let Ω be a frame of discernment. A basic belief assignment (bba) is a function $m : 2^\Omega \rightarrow [0, 1]$ that satisfies $\sum_{A \subseteq \Omega} m(A) = 1$.

The term $m(A)$ is called the basic belief mass (bbm) given to A . The bbm $m(A)$ represents that part of Your belief that supports A , i.e., the fact that the actual world belongs to A , without supporting any more specific subset, by lack of adequate information.

As an example, consider that You learn that the actual world belongs to A , and You know nothing else about its value. Then some part of Your beliefs will be given to A , but no subset of A will get any positive support. In that case, You would have $m(A) > 0$ and $m(B) = 0$ for all $B \neq A$, $B \neq \Omega$, and $m(\Omega) = 1 - m(A)$.

Belief Functions

The bba m does not in itself quantify your belief that the actual world belongs to A . Indeed, the bbm $m(B)$ given to any non empty subset B of A also supports that the actual world belongs to A . Hence, the degree of belief $\text{bel}(A)$ is obtained by summing all the bbms $m(B)$ for all B non empty subset of A . The degree of belief $\text{bel}(A)$ quantifies the total amount of justified specific support given to the fact that the actual world belongs to A . We say justified because we include in $\text{bel}(A)$ only the bbms given to subsets of A . For instance, consider two distinct elements x and y of Ω . The bbm $m(\{x, y\})$ given to $\{x, y\}$ could support x if further information indicates this. However given the available information the bbm can only be given to $\{x, y\}$. We say specific because the bbm $m(\emptyset)$ is not included in $\text{bel}(A)$ as it is given to the subsets that supports not only A but also not A .

The originality of the TBM comes from the non-null masses that may be given to non-singletons of Ω . In the special case where only singletons get positive bbms, the function bel is a probability function. In that last case, the TBM reduces itself to the Bayesian theory.

Shafer assumed $m(\emptyset) = 0$. In the TBM, such a requirement is not assumed. That mass $m(\emptyset)$ reflects

both the non-exhaustivity of the frame and the existence of some conflict between the beliefs produced by the various belief sources.

Expressiveness of the TBM

The advantage of the TBM over the classical Bayesian approach resides in its large flexibility, its ability to represent every state of partial beliefs, up to the state of total ignorance. In the TBM, total ignorance is represented by the vacuous belief function, i.e., a belief function such that $m(\Omega) = 1$, $m(A) = 0$ for all A with $A \neq \Omega$. Hence $\text{bel}(\Omega) = 1$ and $\text{bel}(A) = 0$ for all A strict subset of Ω . It expresses that all You know is that the actual world belongs to Ω . The representation of total ignorance in probability theory is hard to achieve adequately, most proposed solutions being doomed to contradictions. With the TBM, we can of course represent every state of belief, full ignorance, partial ignorance, probabilistic beliefs, or even certainty ($m(A) = 1$ corresponds to A is certain).

Example

Let us consider a somehow reliable witness in a murder case who testifies to You that the killer is a male. Let 0.7 be the reliability You give to the testimony (0.7 is the probability, the belief that the witness is reliable). Suppose furthermore that a priori You have an equal belief that the killer is a male or a female.

A classical probability analysis would compute the probability $P(M)$ of $M =$ "the killer is a male" given the witness testimony as: $P(M) = P(M|\text{Reliable})P(\text{Reliable}) + P(M|\text{Not Reliable})P(\text{Not Reliable}) = 1.0 \times 0.7 + 0.5 \times 0.3 = 0.85$, where "Reliable and Not Reliable refer to the witness" reliability. The value 0.85 is the sum of the probability of M given the witness is reliable (1.) weighted by the probability that the witness is reliable (0.7) plus the probability of M given the witness is not reliable (0.5, the proportion of males among the killers) weighted by the probability that the witness is not reliable (0.3).

The TBM analysis is different. You have some reason to believe that the killer is a male, as so said the witness. But this belief is not total (maximal) as the witness might be wrong. The 0.7 is the belief You give to the fact that the witness tells the truth (is reliable), in which case the killer is male. The remaining 0.3 mass is given to the fact that the witness is not really telling

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