DS/AHP

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INTRODUCTION

DS/AHP was introduced in Beynon, Curry, and Morgan (2000) and Beynon (2002a), and is a nascent method of multi-criteria decision support. Following a hierarchical decision structure, similar to the analytic hierarchy process—AHP (Saaty, 1980), the identification of groups of alternatives (DAs) against the whole set of alternatives considered, over a number of different criteria, is operationalised using Dempster-Shafer theory—DST (Dempster, 1968; Shafer, 1976). As such, the utilisation of DS/AHP means the decision making is embedded with the realisation of the presence of ignorance and non-specificity in the decision judgements made (see Beynon, 2005).

Studies have acknowledged that the making of decisions is often difficult because of uncertainty and conflict in the judgement making process (Shafir, Simonson, & Tversky, 1993). Nutt (1998) references that a key incumbent in decision making is complexity, which prompts difficult decisions as well as manifesting how daunting a decision may appear. The notion of ignorance prevalent within DS/AHP, from DST, has been described as a general term for incompleteness, imprecision, and uncertainty (Smets, 1991), also relating to the subjective imperfection of a decision maker (Motro & Smets, 1997).

The judgement making made by a decision maker, when using DS/AHP, is through the comparisons of identified groups of DAs with respect to their increased belief in preference to that associated with all the DAs considered, over the different criteria. Pertinently, the decision maker does not have to discern between those DAs not preferred and those for which they are ignorant towards. This discernment is undertaken post analysis through the use of belief, plausibility, and pignistic probability measures, which view the presence of ignorance differently. A further measure of judgement making activity is the notion of non-specificity, which with DS/AHP, looks at the level of grouping of preferred DAs (Beynon, 2005b). The homogeneity of the combination process used in DS/AHP has allowed the approach to be effectively applied in a group decision 'consensus building' making environment (Beynon, 2006).

This article presents the rudiments of the DS/AHP technique, as well as a series of example results that exposit its operation in the presence of ignorance, and so forth.

BACKGROUND

For a full description of the fundamentals of the DS/AHP technique, see Beynon (2002a) and Beynon (2005). At various stages within a DS/AHP analysis, the construction of a body of evidence (BOE) is necessary, defined in DST (see Dempster, 1968; Shafer, 1976). A BOE is made up of mass values—$m(\cdot)$. Each $m(\cdot)$: $2^{\Theta} \rightarrow [0, 1]$ is a belief function, such that $m(\varnothing) = 0$ ($\varnothing$ - empty set), and

$$\sum_{x \in 2^{\Theta}} m(x) = 1,$$

where 2^{Θ} denotes the power set of Θ - the frame of discernment (finite set of hypotheses). Any subset x of the Θ for which $m(x)$ is non-zero is called a *focal element* and represents the exact belief in the proposition depicted by x, and $m(\Theta)$ is often interpreted as a level of ignorance, since this weight of evidence is not discernible amongst the hypotheses (Ducey, 2001).

With respect to a single criterion, the judgements made by a decision maker on the considered DAs, when using DS/AHP, are subject to:

a. The preference judgements on identified groups of DAs (focal elements) are with respect to a reference point, in this case all the DAs under consideration
b. Any preference judgements made on groups of DAs are towards the relative belief in the preference of that identified group against the underlying belief associated with the whole set of Das
c. Any DA identified in a group can only appear in one group of DAs

d. Each group of identified DAs should be assigned a unique preference scale value.

The evaluation of the evidence from the judgements made by a decision maker on a criterion, when using DS/AHP, was developed in Beynon (2002a), through the use of a comparison matrix of the form:

$$A_d = \begin{array}{c} \\ s_1 \\ s_2 \\ \vdots \\ s_d \\ \Theta \end{array} \begin{array}{c} \begin{array}{cccccc} s_1 & s_2 & \cdots & s_d & \Theta \end{array} \\ \begin{pmatrix} 1 & 0 & \cdots & 0 & pa_1 \\ 0 & 1 & \ddots & 0 & pa_2 \\ \vdots & \ddots & \ddots & 0 & \vdots \\ 0 & 0 & 0 & 1 & pa_d \\ 1/(pa_1) & 1/(pa_2) & \cdots & 1/(pa_d) & 1 \end{pmatrix} \end{array},$$

where p is the criteria importance value (CIV) for that criterion and, $a_1, a_2, \ldots, a_d$, are the preference scale values assigned to the d groups of DAs, $s_1, s_2, \ldots, s_d$, identified. The sparse matrix A_d shows comparisons are only made with identified groups of alternatives against all those considered, denoted by Θ. Following the process of identifying priority values from comparison matrices in AHP (see Saaty, 1980), the principle right eigenvector associated with the above matrix quantifies the evidence from the included judgements, in the form of a BOE. Moreover, for a criterion c_h, a *criterion* BOE is constructed, defined $m_h(\cdot)$, made up of the mass values:

$$m_h(s_i) = \frac{a_i p}{\sum_{j=1}^{d} a_j p + \sqrt{d}},\ i = 1, 2, \ldots, d$$

and $$m_h(\Theta) = \frac{\sqrt{d}}{\sum_{j=1}^{d} a_j p + \sqrt{d}}.$$

These expressions assign levels of exact belief to the DA groups, $s_1, s_2, \ldots, s_d$, as well as a level of local ignorance ($m_h(\Theta)$). The structure of these expressions indicates the utilised scale values, $a_1, a_2, \ldots, a_d$, need to be positive in nature to assure they form a BOE. These scale values quantify the verbal terms, ranging from moderately to extremely more preferred, used by the DM in their judgement making on groups of DAs compared to all those considered (Beynon et al., 2000).

Within DS/AHP, the setting of scale values began with an adherence to those employed in AHP (including the 1 to 9 scale). However, Beynon (2002a) considered the effect on the range of local ignorance allowed in the judgements made over a single criterion that the scale values available confer. This is a fundamental point within the DS/AHP (in particular DST) since without the presence of ignorance the combination process may exhibit an inhibiting level of conflict (see Murphy, 2000).

The criterion BOEs are independent pieces of evidence, all of which include information on the levels of exact belief in the preferences of groups of DAs. These criterion BOEs need to be combined to construct a final BOE, which includes all the evidence from the criteria on the preference of the DAs. To achieve this final BOE, Dempster's rule of combination is used which allows the combination of independent pieces of evidence. Moreover, considering two BOEs, $m_1(\cdot)$ and $m_2(\cdot)$, the combining function $[m_1 \oplus m_2]: 2^\Theta \rightarrow [0, 1]$, defined by:

$$[m_1 \oplus m_2](y) = \begin{cases} 0 & y = \varnothing \\ \dfrac{\sum_{s_1 \cap s_2 = y} m_1(s_1) m_2(s_2)}{1 - \sum_{s_1 \cap s_2 = \varnothing} m_1(s_1) m_2(s_2)} & y \neq \varnothing \end{cases}$$

is a mass value, where s_1 and s_2 are focal elements associated with the BOEs, $m_1(\cdot)$ and $m_2(\cdot)$, respectively. This combination rule can be used iteratively, to successively combine individual criterion BOE to the previously combined BOEs. The presence of the DAs in the focal elements in the final BOE, defined $m(\cdot)$, depends on their presence in the identified groups of DAs over the different criteria (in the respective criterion BOEs).

How this evidence is used depends on the decision maker, including whether the non-presence of individual DAs in identified groups of preferred DAs is due to their non-preferment or there is a more ignorance based reason. This is a novel viewpoint taken by DS/AHP, since it allows a decision maker to undertake their judgements, without the need to discern between non-preferred DAs and ignorance of DAs (over some or all the criteria). That is, without the allowance for both of these occurrences to exist, it means one may be sacrificed to allow for the other. To take account of