Fuzzy Decision Trees

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INTRODUCTION

The first (crisp) decision tree techniques were introduced in the 1960s (Hunt, Marin, & Stone, 1966), their appeal to decision makers is due in no part to their comprehensibility in classifying objects based on their attribute values (Janikow, 1998). With early techniques such as the ID3 algorithm (Quinlan, 1979), the general approach involves the repetitive partitioning of the objects in a data set through the augmentation of attributes down a tree structure from the root node, until each subset of objects is associated with the same decision class or no attribute is available for further decomposition, ending in a number of leaf nodes.

This article considers the notion of decision trees in a fuzzy environment (Zadeh, 1965). The first fuzzy decision tree (FDT) reference is attributed to Chang and Pavlidis (1977), which defined a binary tree using a branch-bound-backtrack algorithm, but limited instruction on FDT construction. Later developments included fuzzy versions of crisp decision techniques, such as fuzzy ID3, and so forth (see Ichihashi, Shirai, Nagasaka, & Miyoshi, 1996; Pal & Chakraborty, 2001) and other versions (Olaru & Wehenkel, 2003). The expectations that come with the utilisation of FDTs are succinctly stated by Li, Zhao, and Chow (2006):

Decision trees based on fuzzy set theory combines the advantages of good comprehensibility of decision trees and the ability of fuzzy representation to deal with inexact and uncertain information.

A fuzzy environment is embodied through the utilisation of fuzzy membership functions (MFs), which enable levels of association to the linguistic variable representation of numerical attributes (Kecman, 2001). Indeed, it is the notion of MFs that is a relatively unique feature of fuzzy set theory techniques (Li, Deng, & Wei, 2002), namely they allow linguistic and numerical descriptions of the decision rules identified in the case of FDTs.

The FDT technique employed here was first presented in Yuan and Shaw (1995) and Wang, Chen, Qian, and Ye (2000), and attempts to include the cognitive uncertainties evident in data values. Chen, Sheu, and Liu (2007) suggest cognitive uncertainties and the fuzzy maintenance problem, in particular, may be well-represented by Yuan and Shaw's FDT methodology. A further application that utilised this technique was presented in Beynon, Peel, and Tang (2004), which investigated the capabilities of FDTs to predict the levels of audit fees for companies.

BACKGROUND

In classical set theory, an element (value) either belongs to a certain set or it does not. It follows, the definition of a set can be defined by a two-valued membership function (MF), which takes values 0 or 1, defining membership or non-membership to the set, respectively. In fuzzy set theory (Zadeh, 1965), a grade of membership exists to characterise the association of a value x to a set S. The concomitant MF, defined $\mu_S(x)$, has range [0, 1].

The domain of a numerical attribute can be described by a finite series of MFs that each offers a grade of membership to describe a value x, which form its concomitant fuzzy number (see Kecman, 2001). The finite set of MFs defining a numerical attribute's domain can be denoted a linguistic variable (Herrera, Herrera-Viedma, & Martinez, 2000). Different types of MFs have been proposed to describe fuzzy numbers, including triangular and trapezoidal functions, with Yu and Li (2001) highlighting that MFs may be (advantageously) constructed from mixed shapes, supporting the use of piecewise linear MFs. The functional forms of two piecewise linear MFs, including the type utilised here (in the context of the j^{th} linguistic term T_j^k of a linguistic variable A_k), are given by:

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$$\mu_{T_j^k}(x)\begin{cases} 0 & \text{if } x \le \alpha_{j,1} \\ \dfrac{x-\alpha_{j,1}}{\alpha_{j,2}-\alpha_{j,1}} & \text{if } \alpha_{j,1} < x \le \alpha_{j,2} \\ 1 & \text{if } x = \alpha_{j,2} \\ 1-\dfrac{x-\alpha_{j,2}}{\alpha_{j,3}-\alpha_{j,2}} & \text{if } \alpha_{j,2} < x \le \alpha_{j,3} \\ 0 & \text{if } \alpha_{j,3} < x \end{cases}$$

and

$$\mu_{T_j^k}(x)\begin{cases} 0 & \text{if } x \le \alpha_{j,1} \\ 0.5\dfrac{x-\alpha_{j,1}}{\alpha_{j,2}-\alpha_{j,1}} & \text{if } \alpha_{j,1} < x \le \alpha_{j,2} \\ 0.5+0.5\dfrac{x-\alpha_{j,2}}{\alpha_{j,3}-\alpha_{j,2}} & \text{if } \alpha_{j,2} < x \le \alpha_{j,3} \\ 1 & \text{if } x = \alpha_{j,3} \\ 1-0.5\dfrac{x-\alpha_{j,3}}{\alpha_{j,4}-\alpha_{j,3}} & \text{if } \alpha_{j,3} < x \le \alpha_{j,4} \\ 0.5-0.5\dfrac{x-\alpha_{j,4}}{\alpha_{j,5}-\alpha_{j,4}} & \text{if } \alpha_{j,4} < x \le \alpha_{j,5} \\ 0 & \text{if } \alpha_{j,5} < x \end{cases}$$

with the respective *defining values* in list form, [$\alpha_{j,1}$, $\alpha_{j,2}$, $\alpha_{j,3}$] and [$\alpha_{j,1}$, $\alpha_{j,2}$, $\alpha_{j,3}$, $\alpha_{j,4}$, $\alpha_{j,5}$]. Visual representations of these MF definitions are presented in Figure 1, which elucidate their general structure along with the role played by the sets of defining values.

The general forms of MFs presented in Figure 1 ($\mu_{T_j^k}(\cdot)$) shows how the value of a MF is constrained within 0 and 1. In Figure 1*a*, a regularly used triangular MF is shown, based on only three defining values, whereas a more piecewise form is shown in Figure 1b, which requires five defining values. The implication of the defining values is also illustrated, including the idea of associated support (the domains [$\alpha_{j,1}$, $\alpha_{j,3}$] in Figure 1a and [$\alpha_{j,1}$, $\alpha_{j,5}$] in Figure 1b). Further, the notion of dominant support can also be considered where a MF is most closely associated with an attribute value, the domain [$\alpha_{j,2}$, $\alpha_{j,4}$] in Figure 1b.

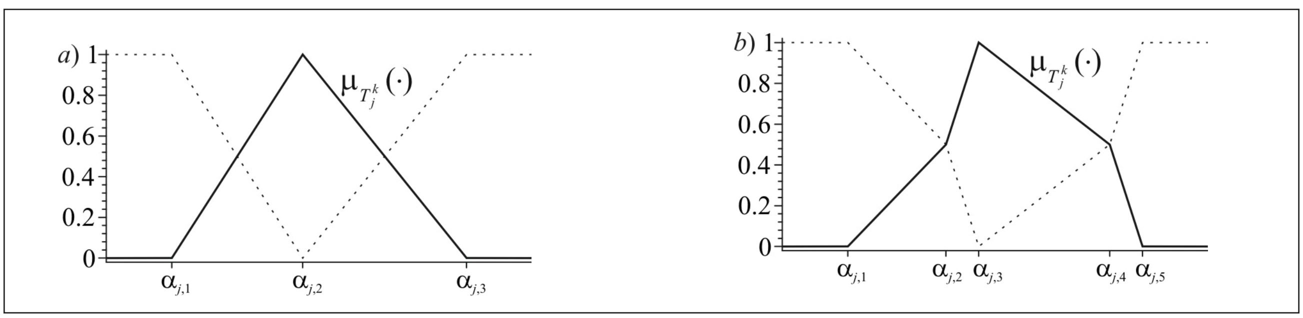
The FDT approach considered here was introduced in Yuan and Shaw (1995) and Wang et al. (2000), which focuses on the minimization of classification ambiguity in the presence of fuzzy evidence. Underlying knowledge related to a decision outcome can be represented as a set of fuzzy '*if* .. *then* ..' decision rules, each of the form:

If (A_1 is $T_{i_1}^1$) and (A_2 is $T_{i_2}^2$) … and (A_k is $T_{i_k}^k$) then C is C_j,

where $A = \{A_1, A_2, .., A_k\}$ and C are linguistic variables in the multiple antecedents (A_is) and consequent (C) statements, respectively, and $T(A_k) = \{T_1^k, T_2^k, .. T_{S_i}^k\}$ and $\{C_1, C_2, \ldots, C_L\}$ are their linguistic terms. Each linguistic term T_j^k is defined by the MF $\mu_{T_j^k}(x)$, which transforms a value in its associated domain to a grade of membership value to between 0 and 1. The MFs, $\mu_{T_j^k}(x)$ and $\mu_{C_j}(y)$, represent the grade of membership of an object's antecedent A_j being T_j^k and consequent C being C_j, respectively.

A MF $\mu(x)$ from the set describing a fuzzy linguistic variable Y defined on X can be viewed as a possibility distribution of Y on X, that is $\pi(x) = \mu(x)$, for all $x \in X$ (also normalized so $\max_{x\in X} \pi(x) = 1$).

Figure 1. General definition of two types of MFs (including defining values)



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