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Uncertainty and Vagueness Concepts in Decision Making

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INTRODUCTION

One of the main challenges in decision making is how to deal with uncertainty and vagueness. The classic uncertainty concept is probability, which goes back to the 17th century. Possible candidates for the title of father of probability are Bernoulli, Laplace, and Pascal. Some 40 years ago, Zadeh (1965) introduced the concept of fuzziness, which is sometimes interpreted as one form of probability. However, we will show that the terms fuzzy and probability are complementary. Recently, in the beginning of the '80s, Pawlak (1982) suggested rough sets to manage uncertainties.

The objective of this article is to give a basic introduction into probability, fuzzy set, and rough set theory and show their potential in dealing with uncertainty and vagueness.

The article is structured as follows. In the next three sections we will discuss the basic principles of probability, fuzzy sets, and rough sets, and their relationship with each other. The article concludes with a short summary.

BACKGROUND

The relationships between these three concepts have been intensively discussed, for example, probability vs. fuzziness (Zadeh, 1995) and fuzziness vs. roughness (Dubois & Prade, 1990; Thiele, 1997; Pawlak, 1985); see Pawlak (2004) for a rough set theory perspective.

Basically, they are defined as follows:

Probability. Probability deals with the uncertainty of whether an event occurs or not. For example, what are the chances of winning the first prize in a lottery? Note, in probability theory the event itself is unambiguously defined.

However, it is uncertain if this event occurs or not. Probability theory is the dominate approach to deal with uncertainty and is used and applied in virtually any area.

Fuzziness. Sometime fuzziness is confused with or even considered as one form of probabilistic uncertainty. However, in fuzzy set theory there is no probabilistic uncertainty about whether an event occurs or not. The so-called fuzzy membership degrees indicate how similar an event is to one or more reference objects. For example, given the two categories *big prize* and *small prize*, a first prize of US$ 1 million would surely be considered predominantly as a big prize and only to a small degree as small prize. So, fuzzy set theory is a concept to describe vagueness in linguistic expressions (big prize, small prize) rather than dealing with probabilistic uncertainty.

After its breakthrough in the beginning of the '90s, fuzzy set theory gained attention in control engineering (fuzzy control) and decision theory and operations research besides others. Fuzzy control has been proven successful in dealing with highly nonlinear systems (Kacprzyk, 1997) while many concepts in decision theory and operations research have been enriched by fuzziness (Zimmermann, 1987).

Roughness. Rough set theory addresses the determination and management of the right amount of information to solve a problem. If there is too much information, it provides concepts to filter out the unnecessary information. In the case of missing information, it groups objects in between sets to show their uncertain membership to one of these sets.

Rough concepts have gained increasing attention in the fields of data mining (Lin & Cercone, 2004), decision making (Komorowski, Pawlak, Polkowski, & Skowron, 1999), and many others.

Since these vagueness concepts are complementary hybrid approaches combining probability, fuzzy sets and rough sets have been developed. For example, the reader is referred to Pal and Skowron (1999) for rough-fuzzy hybridization, to Buckley (2006) for fuzziness and probability, and to Y. Y. Yao (2003) for a probabilistic-rough perspective.

FUNDAMENTALS OF PROBABILITY, FUZZY SETS, AND ROUGH SETS, AND THEIR RELATIONSHIP WITH EACH OTHER

Probability

Probability theory is based on the famous Kolmogorov axioms (Kolmogorov, 1950). Various interpretations of probability have been developed that suit different needs in real-life situations. Furthermore, different degrees of uncertainty have been presented to better deal with probability situations.

Kolmogorov Axioms

The Kolmogorov axioms (Kolmogorov, 1950) are the basis for probability theory. With $E=\{e_1; e_2; \ldots; e_N\}$ being a set of N events and $p(e_j)$ the probability of event e_j ($j=1,\ldots,N$), they are defined as follows:

The probability of an event e_j is nonnegative: $p(e_j) \geq 0$.

The sum over all possible events in E equals 1: $p(E) = 1$.

If the events e_j and e_k are mutually exclusive, then $p(e_j \text{ .or. } e_k) = p(e_j) + p(e_k)$.

Concepts of Probability

Besides the Kolmogorov axioms, various interpretations of probability have been developed, for example (Hájek, 2003), (a) classical probability, (b) frequency interpretation, (c) subjective probability, (d) logical probability, and (e) propensity interpretation. We briefly introduce the first three interpretations (Eisenführ, 1991); the reader is referred to Hájek for an interpretation of the remaining ones.

The classical probability interpretation goes back to the beginnings of probability theory in the 17th century. It is also known as the indifference principle.

As long as there are no reasons given, each event of the set $E=\{e_1; e_2; \ldots; e_N\}$ will occur with the same probability: $p(e_1) = p(e_2)= \ldots = p(e_N) = 1/N$.

Let us consider the example of rolling dice. A probability of 1/6 is assumed for the occurrence of each number as long as there is no evidence for loaded dice.

The frequency interpretation of probability is based on experiments. An experiment is repeated identically N times with N being a reasonable high number. Then the number of experiments with the same outcome is divided by N to obtain its frequency probability. The frequency interpretation of probability is often applied in sciences.

For example, when a suspicion arises that the dice is loaded, then the dice will be rolled many times, for example, N=600. If the dice is not loaded, each number will occur around 100 times in the experiment. The frequency probability is 1/6.

The subjective interpretation is based on personal beliefs. The probabilities are degrees of confidence that an event occurs. A crucial precondition for the application of subjective probabilities is the rationality of the person that estimates the probabilities.

Degrees of Uncertainty

Besides probability interpretations, different degrees of uncertainty can be defined (Eisenführ, 1991): (a) certainty, (b) risk, (c) partial uncertainty, (d) uncertainty, and (e) a game situation.

Certainty implies perfect information. An experiment has one and only one defined outcome; it will occur with a probability of p=1. Under risk, an experiment has more than one possible outcome and the probability of each outcome is known (e.g., $p_A=0.4$ and $p_B=0.6$). A situation under partial uncertainty equals the situation under risk. However, the probabilities of the outcomes are only partially unknown; for instance, only probability intervals can be determined for each outcome (for example, $p_A>0.4$ and $p_B<0.9$). Under uncertainty, no information on the probabilities of the outcomes is given ($p_A=?$ and $p_B=?$). In a game situation, the outcome is influence by one or more opponents.

Fuzzy Set Theory

Fuzzy set theory was introduced by Zadeh in 1965. It is regarded as a central concept of soft computing, which also includes technologies like neural nets and rough sets besides others. In contrast to classic dual set theory, an object must not be assigned to one and only one set but is a member of several sets simultaneously. In the late 1970s, Zadeh (1978) suggested a new branch of fuzzy set theory dealing with possibilities.