Design and Applications of Digital Filters

Gordana Jovanovic Dolecek
INSTITUTE INAOE, Puebla, Mexico

INTRODUCTION

Digital signal processing (DSP) is an area of engineering that "has seen explosive growth during the past three decades" (Mitra, 2005). Its rapid development is a result of significant advances in digital computer technology and integrated circuit fabrication (Jovanovic Dolecek, 2002; Smith, 2002). Diniz, da Silva, and Netto (2002) state that "the main advantages of digital systems relative to analog systems are high reliability, suitability for modifying the system's characteristics, and low cost".

The main DSP operation is digital signal filtering, that is, the change of the characteristics of an input digital signal into an output digital signal with more desirable properties. The systems that perform this task are called digital filters. The applications of digital filters include the removal of the noise or interference, passing of certain frequency components and rejection of others, shaping of the signal spectrum, and so forth (Ifeachor & Jervis, 2001; Lyons, 2004; White, 2000).

Digital filters are divided into finite impulse response (FIR) and infinite impulse response (IIR) filters. FIR digital filters are often preferred over IIR filters because of their attractive properties, such as linear phase, stability, and the absence of the limit cycle (Diniz, da Silva & Netto, 2002; Mitra, 2005). The main disadvantage of FIR filters is that they involve a higher degree of computational complexity compared to IIR filters with equivalent magnitude response (Mitra, 2005; Stein, 2000).

For example let us consider an FIR filter of length $N = 11$ with impulse response

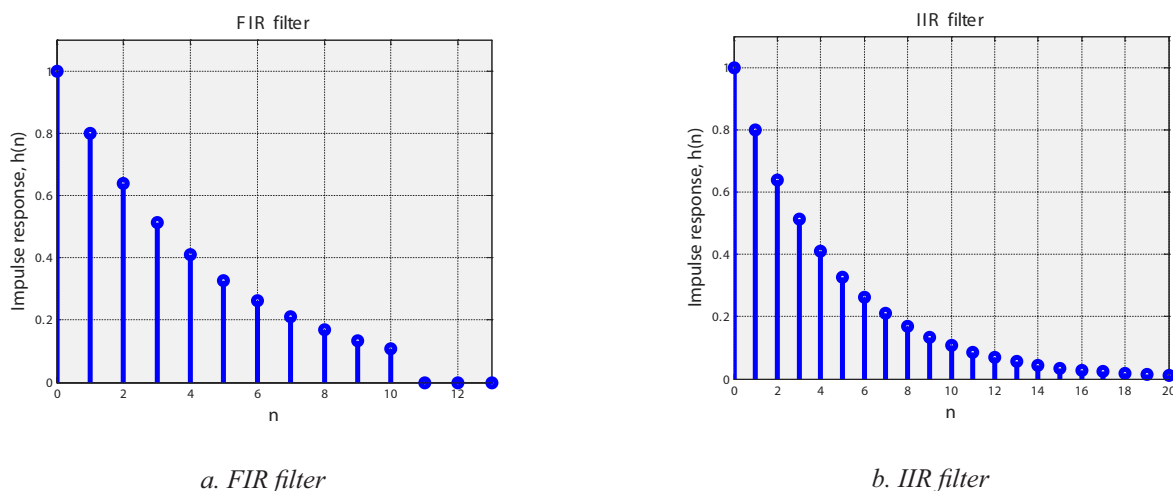$$h(n) = \begin{cases} 0.8^n & for & 0 \leq n \leq 10 \\ 0 & otherwise \end{cases}, \qquad (1)$$

as shown in Figure 1a.

In Figure 1b the initial 20 samples of the impulse response of an IIR filter

$$h(n) = \begin{cases} 0.8^n & for & 0 \leq n \\ 0 & for & n < 0 \end{cases}. \qquad (2)$$

are plotted.

Figure 1. Impulse responses of FIR and IIR filters

a. FIR filter

b. IIR filter

Equation 3.

$$y(n) = x(n) * h(n) = h(n) * x(n) = \sum_k h(k)x(n-k) = \sum_k x(k)h(n-k)$$

BACKGROUND

Digital Filters in Time and Transform Domain

The operation in time domain which relates the input signal $x(n)$, impulse response $h(n)$ and the output signal $y(n)$, is called the *convolution*, and is defined in Equation 3.

The output $y(n)$ can also be computed recursively using the following *difference equation* (Mitra 2005; Proakis & Ingle, 2003),

$$y(n) = \sum_{k=0}^{M} b_k x(n-k) + \sum_{k=1}^{N} a_k y(n-k), \quad (4)$$

where $x(n-k)$ and $y(n-k)$ are input and output sequences $x(n)$ and $y(n)$ delayed by k samples, and b_k and a_k are constants. The order of the filter is given by the maximum value of N and M. The first sum is a *nonrecursive*, while the second sum is a *recursive* part. Typically, FIR filters have only non-recursive part, while IIR filters always have the recursive part. As a consequence, FIR and IIR filters are also known as nonrecursive and recursive filters, respectively.

From (3) we see that the principal operations in a digital filter are multiplications, delays and additions. Using equation (3) we can draw the structure of the digital filter which is also known as a *Direct form* and is shown in Figure 2. More details about filter structures can be found for example in Mitra (2005).

The representation of digital filters in the transform domain is obtained using the *Fourier transform* and *z-transform*.

The Fourier transform of the signal $x(n)$ is defined as

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x(n)e^{j\omega n}, \quad (5)$$

where ω is digital frequency in radians and $e^{j\omega n}$ is an exponential sequence. In general case, the Fourier transform is a complex quantity.

The convolution operation becomes multiplication in the frequency domain,
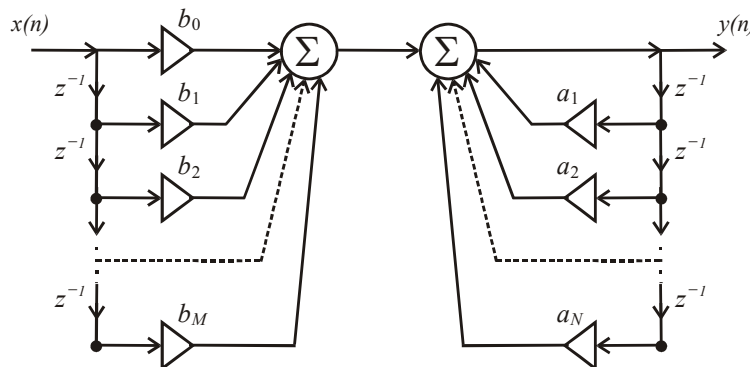
$$Y(e^{j\omega}) = X(e^{j\omega})H(e^{j\omega}), \quad (6)$$

where $Y(e^{j\omega})$, $X(e^{j\omega})$, and $H(e^{j\omega})$, are Fourier transforms of $y(n)$, $x(n)$ and $h(n)$, respectively. The quantity $H(e^{j\omega})$ is called the *frequency response* of the digital filter, and it is a complex function of the frequency ω with a period 2π. It can be expressed in terms of its real and imaginary parts, $H_R(e^{j\omega})$ and $H_I(e^{j\omega})$ or in terms of its magnitude $|H(e^{j\omega})|$ and phase $\varphi(\omega)$,

$$H(e^{j\omega}) = H_R(e^{j\omega}) + jH_I(e^{j\omega}) = |H(e^{j\omega})|e^{j\phi(\omega)}. \quad (7)$$

The amplitude $|H(e^{j\omega})|$ is called the *magnitude response* and the phase $\varphi(\omega)$ is called the *phase response* of the digital filter. For a real impulse response digital filter, the magnitude response is an even function of ω, while the phase

Figure 2. Direct form structure

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