On a Design of Narrowband FIR Low-Pass Filters

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INTRODUCTION

Stearns and David (1996) states that "for many diverse applications, information is now most conveniently recorded, transmitted, and stored in digital form, and as a result, digital signal processing (DSP) has become an exceptionally important modern tool." Typical operation in DSP is digital filtering. Frequency selective digital filter is used to pass desired frequency components in a signal without distortion and to attenuate other frequency components (Smith, 2002; White, 2000). The pass-band is defined as the frequency range allowed to pass through the filter. The frequency band that lies within the filter stop-band is blocked by the filter and therefore eliminated from the output signal. The range of frequencies between the pass-band and the stop-band is called the transition band and for this region no filter specification is given.

Digital filters can be characterized either in terms of the frequency response or the impulse response (Diniz, da Silva & Netto, 2002). Depending on its frequency characteristic, a digital filter is either low-pass, high-pass, band-pass, or band-stop filters. A low-pass (LP) filter passes low frequency components to the output, while eliminating high-frequency components. Conversely, the high-pass (HP) filter passes all high-frequency components and rejects all low-frequency components. The band-pass (BP) filter blocks both low- and high-frequency components while passing the intermediate range. The band-stop (BS) filter eliminates the intermediate band of frequencies while passing both low- and high-frequency components.

In terms of their impulse responses digital filters are either infinite impulse response (IIR) or finite impulse response (FIR) digital filters. Each of four types of filters (LP, HP, BP, and BS) can be designed as an FIR or an IIR filter (Ifeachor & Jervis, 2001; Mitra, 2005; Oppenheim & Schafer, 1999).

The design of a digital filter is carried out in three steps (Ingle & Proakis, 1997):

- Define filter specification
- Approximate given specification
- Implement digital filter in hardware or software.

The topic of filter design is concerned with finding a magnitude response (or, equivalently, a gain) which meets the given specifications. These specifications are usually expressed in terms of the desired pass-band and stop-band edge frequencies ω_p and ω_s, the permitted deviations in the pass-band (pass-band ripple) R_p, and the desired minimum stop-band attenuation A_s, (Mitra, 2005). Figure 1 illustrates a typical magnitude specification of a digital low-pass filter.

In many applications it is often advantageous to employ FIR filters, since they can be designed with exact linear phase, and exibits no stability problems. However FIR filters have a computationally more intensive complexity compared to IIR filters. During past several years, many design methods have been proposed to reduce complexity of the FIR filters, (Chen, Chang and Vinod, 2006; Jovanovic-Dolecek & Mitra, 2007; Lian and Yang, 2001; Rodrigues & Pai, 2005; Yang and Lian, 2003;Yang and Lian, 2006; Zou & Saramaki, 2004).

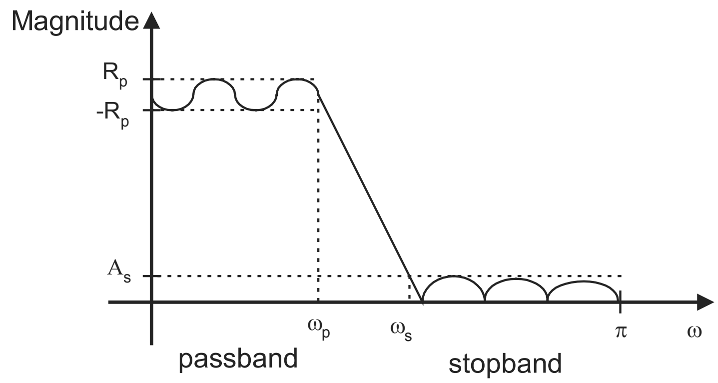
This chapter describes a class of digital filters, called IFIR (interpolated finite impulse response filters), that can implement narrowband low-pass FIR filters with a significantly reduced computational complexity.

The IFIR filter $H(z)$ is a cascade of two filters, an expanded shaping or model filter $G(z^M)$ and an interpolator or image suppressor $I(z)$. In this manner, the narrowband FIR prototype filter $H(z)$ is designed using lower order filters, $G(z)$ and $I(z)$. For more details on the IFIR structure see Neuvo, Cheng, and Mitra (1984) and Jovanovic Dolecek (2003).

An increase in the interpolation factor results in the increase of the interpolation filter order as well as in the decrease of the shaping filter order.

The proposal in Jovanovic Dolecek and Diaz-Carmona (2003) decreases the shaping filter order as much as possible, and efficiently implements the high order interpolator filter. To do so, in Jovanovic Dolecek and Diaz-Carmona (2005) the use of a sharpening recursive running sum (RRS) is proposed as an interpolator in the IFIR structure. The transfer function of an RRS filter with length M is given by

Figure 1. Low-pass filter magnitude specification



$$H_{RRS}(z)=\left(\frac{1}{M}\sum_{k=0}^{M-1} z^{-k}\right)^{L}=\left(\frac{1}{M}\frac{1-z^{-M}}{1-z^{-1}}\right)^{L} \qquad (1)$$

where L is the number of stages. The RRS filter is a linear-phase low-pass filter that requires no multipliers and two additions per output sample, but it has a high pass-band droop and a low stop-band attenuation. The filter sharpening technique, (Hartnett & Boudreaux, 1995; Kaiser & Hamming, 1984; Samadi, 2000), can be used to improve the frequency characteristic of the RRS filter.

We describe here how to find the interpolation factor M, and the number of the stages L, for a given sharpening polynomial, so that the specifications are satisfied with as low as possible complexity.

BACKGROUND

Filter Sharpening

The sharpening technique, which was first proposed by Kaiser and Hamming (1984), attempts to improve both the pass-band and stop-band of a linear FIR filter by using multiple copies of the same filter. This technique is based on the use of a polynomial approximation which maps a transfer function before sharpening, to a new transfer function. The sharpening polynomial is obtained using the closed formula proposed by Samadi (2000). The polynomial coefficients are always integers, which can be implemented as Shift-and-Add multipliers (Parhi, 1999).

Algorithm Description

This section exposes the algorithm for computing the main design parameters for the design of a narrowband low-pass filter using an IFIR structure, where the interpolator filter is the sharpening recursive running (RRS) filter.

The algorithm is based on an iterative design of the model filter G(z) using following specifications:

$$\omega_{Gp}=M\omega_p,\ \ \omega_{Gs}=M\omega_s,$$
$$\delta_{Gp}=\alpha\,\delta_p,\ \ \delta_{Gs}=\delta_s, \qquad (2)$$

where ω_{Gp} and ω_{Gs} are the normalized pass-band and stop-band frequencies, respectively, δ_{Gp} the maximum pass-band ripple and δ_{Gs} the minimum stop-band attenuation of the model filter $G(z)$, and ω_p, ω_s, δ_p and δ_s are the corresponding design specifications of the desired filter. The parameter α is a key factor in this design. The higher value of α the smaller model filter order, and vice versa. For a given value of M the maximum value of α is determined by the sharpened RRS filter pass-band droop. Our experience says that it is useful to take the values of α in the range $0.5 \le \alpha \le 0.9$.

In Figure 2 is shown the flow diagram of the algorithm. At first step, the maximum possible interpolation factor, $M = M_{max} = \pi/\omega_s$ and the minimum possible number of RRS stages $L = 1$, are selected. The choice of M is a key factor to satisfy the pass-band specification, while the choice of L is a key factor to satisfy the stop-band specification of the designed overall filter. Consequently, in the next steps the factor M is increased until the pass-band specification is satisfied, interpolation factor searching loop, while the parameter L is increased until the stop-band specification is reached, RRS stages searching loop. If as a result, an even