

# Efficient Multirate Filtering

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## INTRODUCTION

Efficient multirate filters have been developed during the past three decades for implementation of digital filters with stringent spectral constraints (Crochiere & Rabiner, 1981, 1983; Ansari & Liu, 1993; Bellanger, 1984, 1989; DeFata, Lucas & Hodgkiss, 1988; Vaidyanathan, 1990, 1993; Fliege, 1994; Zelniker & Taylor, 1994; Proakis & Manolakis, 1996; Mitra, 2001; Milić & Lutovac, 2002; Hentschel, 2002).

A multirate filter can be defined as a digital filter in which the input data rate is changed in one or more intermediate points. With the efficient multirate approach, computations are evaluated at the lowest possible sampling rate, thus improving the computational efficiency, increasing the computation speed, and lowering the power consumption. Multirate filters are of essential importance for communications, digital audio, and multimedia.

## MULTIRATE FILTERING TECHNIQUES

Multirate filtering is one of the best approaches for solving complex filtering problems when a single filter operating at a fixed sampling rate is of a very high order. With a multirate filter, the number of arithmetic operations

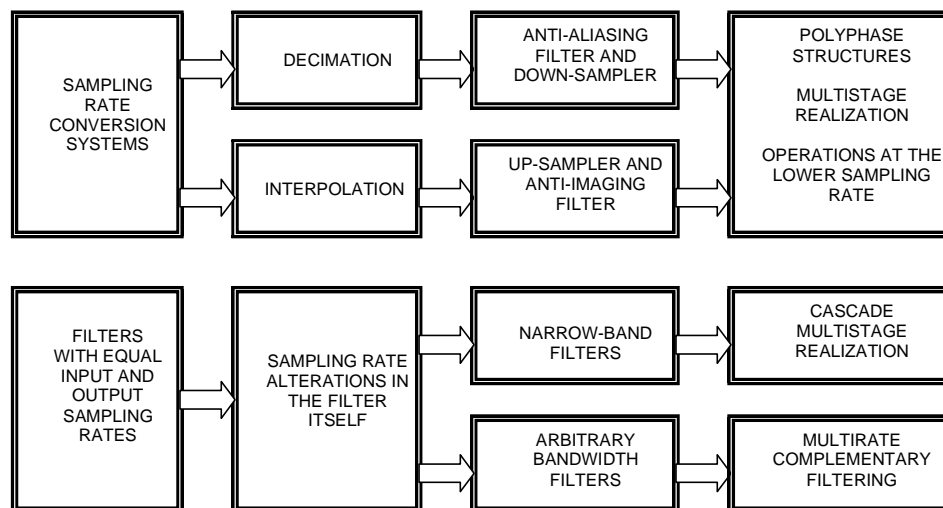
per second is considerably reduced. The multirate technique is used in filters for sampling rate conversion where the input and output rates are different, and also in constructing filters with equal input and output rates. For multirate filters, FIR (finite impulse response) or IIR (infinite impulse response) transfer functions can be used. An FIR filter easily achieves a strictly linear phase response, but requires a larger number of operations per output sample when compared with an equal magnitude response IIR filter. Multirate techniques significantly improve the efficiency of FIR filters that makes them very desirable in practice.

Figure 1 depicts an overview of different multirate filtering techniques.

## Polyphase Implementation

Polyphase realization is used to provide an efficient implementation of multirate filters. A polyphase structure is obtained when an  $N$ th order filter transfer function is decomposed into  $M$  polyphase components,  $M < N$ . For FIR filters, polyphase decomposition is obtained simply by inspection of the transfer function (Crochiere & Rabiner, 1983; Vaidyanathan, 1993; Fliege, 1994; Proakis & Manolakis, 1996; Mitra, 2001). For multirate IIR filters, several approaches to polyphase decomposition have been developed (Bellanger, Bonnerot & Coudreuse, 1976;

Figure 1. An overview of multirate filtering techniques



Crochiere & Rabiner, 1983; Drews & Gaszi, 1986; Renfors & Saramäki, 1987; Russel, 2000; Krukowski & Kale, 2003).

## Filters for Sampling Rate Conversion

Filters are used in decimation to suppress aliasing and in interpolation to remove imaging. The performance of the system for sampling rate conversion is mainly determined by filter characteristics. Since an ideal frequency response cannot be achieved, the choice of an appropriate specification is the first step in filter design.

Reducing the sampling rate by a factor of  $M$  is achieved by omitting every  $M-1$  sample, or equivalently keeping every  $M$ th sample. This operation is called down-sampling. In order to avoid aliasing, a low-pass anti-aliasing filter before down-sampling is needed. Therefore, a decimator is a cascade of an anti-aliasing filter and a down-sampler. To increase the sampling rate (interpolation by factor  $L$ ),  $L-1$  zeros are inserted between every two samples (up-sampling). An interpolation filter has to be used to prevent imaging in the frequency band above the low-pass cutoff frequency. An interpolator is a cascade of an up-sampler and an anti-imaging filter.

The efficiency of FIR filters for sampling rate conversion is significantly improved using the polyphase realization. Filtering is embedded in the decimation/interpolation process, and a polyphase structure is used to simultaneously achieve the interpolation/decimation by a given factor, but running at a low data rate.

Due to the polyphase multirate implementation, the number of arithmetic operations in linear phase FIR filters is decreased by a factor  $M$  (or  $L$ ). An effective method, which leads to high efficiency for a high-order FIR filter, is proposed in Muramatsu and Kiya (1997). Efficient decimation and interpolation for the factor  $M=2$  ( $L=2$ ) is achieved with FIR half-band filters, since the number of constants is a half of the filter length.

Polyphase IIR filters require lower computation rates among the known decimators and interpolators (Renfors & Saramäki, 1987). If a strictly linear phase characteristic is not requested, an IIR filter is an adequate choice. Moreover, an IIR transfer function can be designed to approximate a linear phase in the pass-band (Jaworski & Saramäki, 1994; Lawson, 1994; Surma-Aho & Saramäki, 1999). An IIR decimator or interpolator is particularly useful in applications that cannot tolerate a considerably large delay of an adequate FIR decimator or interpolator. For a restricted class of filter specifications, an attractive solution based on all-pass sub-filters can be used, leading to very efficient implementation (Renfors & Saramäki, 1987; Krukowski & Kale, 2003). The most attractive solution is an IIR half-band filter implemented with two all-pass sub-filters (Renfors & Saramäki, 1987; Johansson &

Wanhammar, 1999; Milić & Lutovac, 2002; Krukowski & Kale, 2003). For a rational conversion factor  $L/M$ , a very efficient decomposition of IIR filter is proposed in Russel (2000).

## Filters with Equal Input and Output Rates

Digital filters with sharp transition bands are difficult, sometimes impossible, to be implemented using conventional structures. A serious problem with a sharp FIR filter is its complexity. The FIR filter length is inversely proportional to transition—width and complexity becomes prohibitively high for sharp filters (Crochiere & Rabiner, 1983; Vaidyanathan, 1993; Saramäki, 1993; Fliege, 1994; Proakis & Manolakis, 1996; Mitra, 2001). In a very long FIR filter, the finite word-length effects produce a significant derogation of the filtering characteristics in fixed-point implementation (Mitra, 2001). IIR filters with sharp transition bands suffer from extremely high sensitivities of transfer function poles that make them inconvenient for fixed-point implementation (Lutovac, Tošić & Evans, 2000). In many practical cases, the multirate approach is the only solution that could be applied for the implementation of a sharp FIR or IIR filter. Thus to design a multirate narrowband low-pass FIR or IIR filter, a classic time-invariant filter is replaced with three stages consisting of: (1) a low-pass anti-aliasing filter and down-sampler, (2) a low-pass kernel filter, and (3) an up-sampler and low-pass anti-imaging filter (Crochiere & Rabiner, 1983; Fliege, 1994; Mitra, 2001; Milić & Lutovac, 2002). The total number of coefficients in a multirate solution is considerably lower than the number of coefficients of a single-rate time invariant filter.

## Multistage Filtering

For decimation and interpolation filters, and for multirate narrowband filters, additional efficiency may be achieved by cascading several stages, each of them consisting of a sub-filter and down-sampler for decimation and an up-sampler and sub-filter for interpolation (Fliege, 1994; Mitra, 2001; Milić & Lutovac, 2002). Design constraints for sub-filters are relaxed if compared to an overall filter. Hence, by using the multistage approach, the total number of coefficients is significantly reduced when compared with the single stage-design. The effects of finite word-length in sub-filters are low in comparison with the single-stage overall filter. When a decimation/interpolation factor is expressible as a power-of-two, the application of half-band filters improves the efficiency of the system.

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