Chapter 15
Modified Iterative Methods for Solving Fully Fuzzy Linear Systems

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ABSTRACT

In the present chapter, we give an overview of computational iterative schemes for fuzzy system of linear equations. We also consider fully fuzzy linear systems (FFLS) and demonstrate a class of the existing iterative methods using the splitting approach for calculating the solution. Furthermore, the main aim in this work is to design a numerical procedure for improving this algorithm. Some numerical experiments are illustrated to show the applicability of the methods and to show the efficiency of proposed algorithm, we report the numerical results of large-scaled fuzzy problems.

INTRODUCTION

Unfailing real world problems in economics, finance, mechanics etc. can lead to solving a system of linear equations. There are many methods for solving linear systems, see(Barrett et al., 1994; Edalatpanah, 2008; Eisenstat, Elman, & Schultz, 1983; Greenbaum, 1997; Martins, Trigo, & Evans, 2007; Saad, 2003; Saberi Najafi & Edalatpanad, 2013a; Saberi Najafi & Edalatpanah, 2013b, 2013c, 2014b; Saberi Najafi & Edalatpanah, 2011; Varga, 2009; Young, 2014; Zhang, Huang, Cheng, & Wang, 2012) and the references therein. Let us consider the following linear systems

$$Ax=b, \tag{1}$$

However, when the estimation of the system coefficients is imprecise and only some vague knowledge about the actual values of the parameters is available, it may be convenient to represent some or all of them with fuzzy numbers. Fuzzy data is being used as a natural way to describe uncertain data. Fuzzy concept was introduced by Zadeh (Zadeh, 1965, 1972) and following his work, many papers and books were published in fuzzy system theory; see (Bellman & Zadeh, 1970; Bezdek, 2013; Chen, 2000; Cordón, 2001; Driankov, Hellendoorn, & Reinfrank, 2013; Hájek, 1998; Höppner, 1999; Jang & Sun,

DOI: 10.4018/978-1-5225-0914-1.ch015

1995; Kusko, 1993; Pham, Xu, & Prince, 2000; Ragin, 2000; Sheridan, 1992; Wasserman, 1993; Yager & Filev, 1994; Zadeh, 1997; Zimmermann, 2001).We refer the reader to (Kaufmann & Gupta, 1991) for more information on fuzzy numbers and fuzzy arithmetic. Fuzzy systems are used to study a variety of problems including fuzzy metric spaces (Alaca, Turkoglu, & Yildiz, 2006; Gregori, Romaguera, & Veeramani, 2006; J. H. Park, 2004), fuzzy differential equations (Bede & Gal, 2005; Buckley, Eslami, & Feuring, 2002; Kaleva, 1987; Khastan, Nieto, & Rodríguez-López, 2011; Khastan & Rodríguez-López, 2015; Malinowski, 2012; J. Y. Park & Han, 2000), particle physics (El Naschie, 2004a, 2004b), Game theory (Kacher & Larbani, 2008; Larbani, 2009; Maeda, 2000; Oliveira & Petraglia, 2014; Saberi Najafi & Edalatpanah, 2012a; Yang & Gao, 2014), optimization (Amid, Ghodsypour, & O'Brien, 2006; Edalatpanah & Shahabi, 2012; Guua & Wu, 1999; Huang, Baetz, & Patry, 1995; Inuiguchi, Ichihashi, & Kume, 1990; Lee & Li, 1993; Najafi & Edalatpanah, 2013d; Rommelfanger, 2007; Shamooshaki, Hosseinzadeh, & Edalatpanah, 2014; Słowiński, 1986; Yu, 2002), fuzzy linear systems(Allahviranloo, 2004; Asady, Abbasbandy, & Alavi, 2005; Dehghan & Hashemi, 2006b; Dehghan, Hashemi, & Ghatee, 2006, 2007; Dubois & Prade, 1980; Friedman, Ming, & Kandel, 1998; Ma, Friedman, & Kandel, 2000; Nasseri, Sohrabi, & Ardil, 2008; Saberi Najafi & Edalatpanah, 2012b), and so on.

In this chapter, we design a modified iterative method for solving fully fuzzy linear systems (FFLS). This paper is organized as follows:

In Section 2 some basic definitions and arithmetic are reviewed. In Section 3 a background for solution of fuzzy system of linear equations are presented. In Section 4 a new method is proposed for solving FFLS. In section 5 numerical results are considered to show the efficiency of the proposed method. Section 6 ends this chapter with a conclusion.

Some Basic Definition and Arithmetic Operations

In this section, an appropriate brief introduction to preliminary topics such as fuzzy numbers and fuzzy calculus will be introduced and the definition for FFLS will be provided. For details, we refer to (Abdolmaleki & Edalatpanah, 2014b; Dehghan et al., 2006, 2007).

Definition 2.1: Let X denote a universal set. Then a fuzzy subset $\tilde{A}$ of X is defined by its membership function $\mu_{\tilde{A}} : X \to [0,1]$; which assigns a real number $\mu_{\tilde{A}}(x)$ in the interval [0,1], to each element $x \in X$, where the value of $\mu_{\tilde{A}}(x)$ at x shows the grade of membership of x in $\tilde{A}$.

A fuzzy subset $\tilde{A}$ can be characterized as a set of ordered pairs of element x and grade $\mu_{\tilde{A}}(x)$ and is often written $\tilde{A} = \{(x, \mu_{\tilde{A}}(x)); x \in X\}$.The class of fuzzy sets on X is denoted with $\Gamma(X)$.

Definition 2.2 A fuzzy set with the following membership function is named a triangular fuzzy number and in this paper we will use these fuzzy numbers.

$$\mu_{\tilde{A}}(x) = \begin{cases} 1 - \dfrac{m-x}{\alpha}, & m-\alpha \le x \le m, \alpha > 0, \\ 1 - \dfrac{x-m}{\beta}, & m \le x \le m+\beta, \beta > 0, \\ 0, & else. \end{cases}$$