Chapter 36
On Cognitive Foundations and Mathematical Theories of Knowledge Science

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ABSTRACT

Knowledge is one of the fundamental cognitive objects in the brain among those of data, information, and intelligence. Knowledge can be classified into two main categories, i.e., conceptual knowledge for knowing to-be and behavioral knowledge for knowing to-do, particularly the former. This paper presents a basic study on a mathematical theory of knowledge towards knowledge science. The taxonomy and cognitive foundations of knowledge are explored, which reveal that the basic cognitive structure of conceptual knowledge is a formal concept and that of behavioral knowledge is a formal process. Mathematical models of knowledge are created in order to enable formal representation and rigorous manipulation of knowledge. A set of formal principles and properties of knowledge is elicited and elaborated towards the development of knowledge science and cognitive knowledge systems. It is discovered that the basic unit of knowledge is a binary relation, shortly bir, as a counterpart of bit (a binary digit) for information and data.

1. INTRODUCTION

The famous perception on knowledge is Francis Bacon's assertion in 1597 that "Knowledge is power." Knowledge is acquired and comprehended information generated by the brain, which is embodied as a concept and its relations to existing ones. Knowledge can be classified into two main categories as those of conceptual and behavior knowledge [Berkeley, 1710; Boole, 1854; Russell, 1956; Dancy, 1985; Wilson & Keil, 1999; Pojman, 2003; Wang, 2003; 2009c; 2012a; 2012b]. The former are created by knowing to-be such as abstract knowledge and experience, while the latter are acquired by knowing to-do such as behaviors and skills.

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Knowledge science is an emerging field that studies the nature of human knowledge, principles and formal models of knowledge representation, and theories for knowledge manipulations such as creation, generation, acquisition, composition, memorization, retrieval, and depository in knowledge engineering. All scientific, engineering, and humanity disciplines generate and process knowledge. The following survey highlights related disciplines such as philosophy, cognitive science, linguistics, computer science, information science, neuroinformatics, cognitive informatics, and mathematics that contribute to the development of knowledge science.

Studies on knowledge in *philosophy* form the domain of epistemology [Berkeley, 1710; Boole, 1854; Russell, 1956; Dancy, 1985]. Studies on knowledge in *cognitive science* reveal the mechanisms of knowledge acquisition, storage, and retrieval in the brain [Matlin, 1998; Gabrieli, 1998; Wilson & Frank, 1999; Wang, 2002; 2009c; 2012c]. Studies on knowledge in *linguistics* result in syntactic and semantic theories [Chomsky, 1965; Keenan, 1975; Saeed, 2009; Zadeh, 1997; 2004; Wang, 2013b; Wang & Berwick, 2012; 2013], linguistic knowledge bases [Crystal, 1987; Miller, 1995], and cognitive linguistics [Pullman, 1997; Evans & Green, 2006; Wang & Berwick, 2012, 2013]. Studies on knowledge in *computer and information sciences* lead to the establishment of artificial intelligence [Shannon, 1948; Turing, 1950; McCarthy et al., 1955; von Neumann, 1958; Debenham, 1989; Albus, 1991; Gruber, 1993; Brewster et al., 2004; Wang, 2007a; 2010; 2012b; 2012c; 2014b; 2015a; 2015b], and machine learning [Gagne, 1985; Bender, 2000; Wang, 2013b; 2015f]. Studies on knowledge in *neuroinformatics* [Wilson & Keil, 1999; Hampton, 1997; Gabrieli, 1998; Wang, 2003, 2009b, 2012b, 2013a; Wang & Wang, 2006;] deepen the understanding of internal knowledge representation as the object-attribute-relation (OAR) model [Wang, 2007c] and the neural circuit theory for knowledge representation [Wang & Fariello, 2012]. Studies on knowledge in *cognitive informatics* [Wang, 2002, 2003, 2006, 2007b, 2008b, 2009a, 2009b, 2009c; Wang et al., 2006; 2009a; 2009b; 2010] lead to the layered reference model of the brain (LRMB) [Wang et al., 2006] that provides the context of knowledge and learning with the support of other cognitive processes. In order to rigorous explain the framework of knowledge manipulation, a theory of cognitive knowledge base (CKB) [Wang, 2014a] and a cognitive system known as the cognitive learning engine (CLE) [Wang, 2013b; Hu, Wang, & Tian, 2010] are developed. Studies in *mathematics* create a set of denotational mathematical structures [Wang; 2007a; 2007b; 2008a; 2009d; 2012e; 2015e; 2015f; 2016] such as concept algebra [Wang, 2008d; 2015f], semantic algebra [Wang, 2013b], inference algebra [Wang, 2011; 2012d] for rigorous manipulations of formal knowledge in knowledge science and engineering.

In traditional epistemology, knowledge is perceived as a justified true belief [Russell, 1956; Dancy, 1985; Debenham, 1989] in a tripartite pattern, which can be formally expressed [Wang, 2015f] as follows.

Definition 1: The *tripartite form of knowledge* perceives that logical knowledge *k* is a subjective proposition *p* of a person *H* whereas *p iff H* believes *p* and the belief is justified, i.e.:

$$\begin{aligned} &\forall p, \theta, \text{and } H,\ \exists k, \\ &\quad k : p \vdash \mathrm{T} \wedge \mu_H(p \vdash \mathrm{T}) \geq \theta \end{aligned} \tag{1}$$

where $\vdash$ denotes an inference, μ_H the justification of an inference by *H*, θ a threshold of confidence, and *T* the logical constant of true.