Chapter 58
Information Systems on Hesitant Fuzzy Sets

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ABSTRACT

Knowledge extraction from information systems is one of the most significant problems in artificial intelligence. This paper attempts to study information systems in the hesitant fuzzy domain. It studies information systems which has a set of possible membership values. Illustration of a case is provided where the hesitant membership values are arrived at from attribute values whose membership values are a family of sets. The membership value here would turn out to be a subset of the power set of membership values from the usual information system. Although it does not mean that it is arrived at from usual information systems. Reduct, core, relative reduct, relative core and the corresponding indiscernibility matrices are also studied. Apart from these, paper also discusses the homomorphisms between hesitant information systems. For two homomorphic information systems the reduct and core of one information system are the corresponding images of the reduct and core of the other information system under this homomorphism.

1. INTRODUCTION

Extraction of valuable information from data sets is an age old problem with significance in mathematical models relating to artificial intelligence, data mining, cognitive sciences and similar areas. Towards this end various different methods have been used to analyse information systems. Among these methods mathematical models involving fuzzy sets, rough sets and their hybrid versions have been used effectively.

Many mathematical theories have successfully characterised vagueness and imprecision while trying to address issues regarding uncertainty. Fuzzy set theory introduced by Zadeh (1965) has been one of the first and highly successful steps in this direction. Many extensions of fuzzy sets have been developed

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ever since. Intuitionistic fuzzy sets Atanassov (1986), Type 2 fuzzy sets Mizumoto and Tanaka (1976) and Fuzzy multisets Miyamoto (2000) are just some of the most accepted ones.

One of the recent extensions to fuzzy sets is the introduction of Hesitant fuzzy sets by Torra (2010), Torra and Narukawa (2009). Hesitant fuzzy sets help in addressing the issues regarding hesitancy and difficulties in assigning membership values to elements in a set. It allows us to assign a set of possible membership values to any element in the set. Although they are not meant for cases involving membership values with an error margin or a possibility distribution, they are extremely useful in decision making situations where we are hesitant about the right values to the membership functions. Xia and Xu (2011) discuss the relationship between intutionistic fuzzy sets and hesitant fuzzy sets and move on to study various aggregation operators for hesitant fuzzy elements applying them in decision making problems with anonymity. A hesitant fuzzy element is the set of possible membership values of an element in the set. Distance measures, similarity measures and correlation measures of hesitant fuzzy elements and their application in clustering analysis have been studied in Xu and Xia (2011a), Xu and Xia (2011b), Chen, Xu and Xia (2013). Various applications of hesitant fuzzy sets in decision making problems are discussed in Rodriguez, Martinez and Herrera (2012), Gu, Wang and Yang (2011), Wei (2012).

Pawlak (1982) introduced Rough set theory where he attempts at characterising uncertainty by considering the boundary region of a set. This in a way is inspired from Frege's concept of boundary region of a set. Equivalence classes are used to divide the set into its lower and upper approximations. The lower approximation has all those equivalence classes that completely belong to the set and the upper approximation contains those that have a non-empty intersection with the set. The boundary region will consist of the difference between the upper and the lower approximations which if non empty will give us a rough set. Rough sets have been effectively used in classification problems with information systems. Pawlak and Marek (1981), Pawlak (1991) lay some of the foundations towards this direction. Pawlak (1997) gives an outline of various concepts in rough set theory and suggests its possible applications. Kryszkiewicz (1998) has used concepts in rough sets for reasoning in incomplete information systems which may also be used in systems with multivalued attributes. Variable precision rough set model by Ziarko (1993), which in a way is a generalization of rough sets, models classification problems involving uncertain or imprecise information. Some methods for computing optimal decision rules from incomplete systems are analyzed in Kryszkiewicz (1999), Leung, Wu and Zhang (2006), Leung and Li (2003). Leung, Fischer, Wu, and Mi (2008) developed a general framework for mining of classification rules in interval-valued information systems. Dai and Tian (2013) studied fuzzy rough set model for set valued data employing the concept of a fuzzy relation. Wang, Wu, Chen, Hu and Wu (2008) and Wang, Chen and Zhu (2009) have discussed the homomorphism's between fuzzy information systems where attribute reductions of the original system and image system are equivalent to each other. Rough computational methods for extraction of knowledge in the form of classification have been presented in Guan and Bell (1998).

This paper introduces hesitant information systems. Basically they are information systems on a hesitant fuzzy domain. The attribute values corresponding to every element will be a set of values from [0,1]. We arrive at such a hesitant information system (HIS) with an illustrative example. The second section discusses the preliminaries and basic concepts regarding hesitant fuzzy sets. It concludes by referring to an earlier definition of a hesitant fuzzy relation. Hesitant fuzzy information systems are introduced and discussed in section 3. Section 4 introduces the reduct and core of an HIS and studies the discernibility matrix which also helps in ascertaining the core. Section 5 studies these concepts for a hesitant decision system. Here we have two categories of attributes which are the conditional attributes