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The Concept of the Shapley Value and the Cost Allocation Between Cooperating Participants

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INTRODUCTION

Game theory is a branch of applied mathematics that studies strategic situations in which participants (players) act in order to maximize their returns (payoffs). As such, game theory provides models of rational behavior (decision-making) for strategic interactions.

Many types of problems that involve decision strategies for cooperating or non-cooperating participants present a fruitful ground for the application of mathematical game theory (Dowd, 2004; Cachon & Netessine, 2004).

In particular, cost allocation problems arise in many situations in which participants work together, such as healthcare providers who have to coordinate patient care in order to reduce the cost and improve the quality of care. It is demonstrated that a natural framework for developing a methodology for cost allocation problems could be based on game theoretical concepts (Tijs and Driessen, 1986; Roth, 1988; Young, 1994; Moulin, 2003). Several concepts for determining the 'fair' cost allocation have been proposed but only a few of these concepts have been used in practice: the nucleolus and the Shapley value.

In this chapter, these two concepts are illustrated side by side. The focus is on examples of practical application of the Shapley value, specifically in healthcare settings. The following two cases are considered in details: (i) the general application of the Shapley value methodology for cost allocation between cooperating providers of care applied to the bundled payment model mandated recently by the Center for Medicare Services' (CMS), and (ii) an important particular case, in which each participant uses only a portion of the largest participant's asset (the so-called airport game).

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BACKGROUND

By pooling resources and cooperating the participants usually reduce the total joint costs and realize savings. The question arises is how the reduced costs or the realized savings should be fairly allocated between them.

There could be different definitions of fair allocation. Some of them are:

- **Equitable Allocation:** Gives everyone the same satisfaction level, i.e. the proportion each player receives by their own valuation is the same for all of them. This is a difficult aim as players might not be truthful if asked their valuation.
- **Proportional Allocation:** Guarantees that each player gets his share. For instance, if three people divide up an asset then each gets at least a third by their own valuation.
- **Envy-Free Allocation:** Everyone prefers his own share to the others. No one is jealous of anyone else. No one would trade his share with anyone else's.
- **An Efficient or Pareto Optimal Allocation:** Ensures that no other allocation would make someone better off without making someone else worse off. The term efficiency comes from the economics idea of the efficient market.

- **Merit-Based Allocation:** The more one brings to the coalition, the more one gets out of the division of the accumulated gains.

A concept of fairness is rather subjective. It depends on the participants' socio-economic views and other factors. The fairness schemes described in the next sections form a basis of the two most popular cost allocation approaches: the nucleolus (Tijs and Driessen, 1986; Saad, 2009) and the Shapley value (Roth, 1988; Young, 1994).

MAIN FOCUS

The Nucleolus Concept

The nucleolus can be defined as an equilibrium that finds the 'center of gravity' of the so-called core. The core is defined as a set of inequalities that meet the requirement that no participant or a group of participants pays more than their stand-alone cost. The fairness criteria used by the nucleolus is minimizing the maximum "unhappiness" of a coalition. "Unhappiness" (or "excess") of a coalition is defined as the difference between what the members of the coalition could get by themselves and what they actually get if they accept the allocations suggested by the nucleolus.

More formally, an *n*-player game is defined by the set $N = \{1, 2,...,n\}$ and a function $v(*)$, which for any subset gives a number $v(S)$ called the value of S. The characteristic value of the coalition S, denoted by $v(S)$, is the payoff that all players in the coalition S could jointly obtain. Let x_i be a payoff for player $i= 1, 2, …,n$. The nucleolus solution is defined as $x = (x_{1,} x_{2, ...,} x_n)$ such that the excess ("unhappiness") $e_S(x) = v(S) - \sum x_i$ of any possible coalition S cannot be lowered without increasing any other greater excess. With this definition, the nucleolus is a solution that makes the largest "unhappiness" of the coalitions as small as possible.

There is no general closed-form formula for the nucleolus calculation, except for the recently developed analytic solution for a particular three-player case (Leng and Parlar, 2010). In general, the nucleolus has to be computed numerically in an iterative manner by solving a series of linear programming (LP) problems, or by solving a very large-scale LP problem. More specifically, the linear programming problem formulation is (Saad et al, 2009):

Z -> min

subject to:

$$Z \sum_{i \in S} x_i \geq v(S) \qquad (1)$$

$$\sum_{i \in N} x_i = v(N) \qquad (2)$$

The advantage of the nucleolus is that it always exists and that it is unique for all non-empty cores. Therefore, some researchers have used this concept to analyze business and management problems. As an early application of the nucleolus concept, Barton (1992) suggested the nucleolus solution as the mechanism to allocate joint costs among entities who share a common resource. At the same time, due to the complexity of the calculations for large coalitions, the nucleolus has not been extensively used to solve the various allocation-related problems.

Another problem with the nucleolus is that it does not exhibit the monotonicity property (Tijs and Driessen, 1986). Cost allocation concepts that do not exhibit monotonicity could result in having some members paying less if the total cost increases or having paid more if the total cost decreases. An example of a water supply project is available in which cost overrun would actually benefit some participants if the nucleolus method is used for allocating costs: the higher total project cost results in lowering contributions of some participants (Young et al, 1982). Thus, if regulatory agencies do not have the means of monitoring