Preferences, Utility, and Stochastic Approximation

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INTRODUCTION

A complex system with human participation like "human-process" is characterized with active assistance of the human in the determination of its objective and description, and in decision-taking during its development. The construction of a mathematically grounded model of such a system is faced with the problem of shortage of mathematical precise information that presents the human activity. Often, in complex processes there is a lack of measurements or even clear scales for the basic heuristic information. On this level of investigations the decisions is close to the art to choose the right decision among great number of circumstances and often without associative examples of similar activity. The basic common source of information here are the expressed human preferences. A solution of this problem is to seek interpretation and expression of different aspects of the complex system through expert analysis and description of the expert's preferences as an element of the system.

The presentation of human preferences analytically with utility functions is a good possible approach for their mathematical description. It is the first step in realization of a human-centered value driven design process and decision making, whose objective is to avoid the contradictions in human decisions and to permit mathematical calculations. In this approach the human being has the role of decision maker (DM). If the subjective and probability uncertainty of DM preferences is interpreted as some stochastic noise, stochastic programming can be used for recurrent evaluation of the utility function in the sense of von Neumann with noise (uncertainty) elimination. In fact this is pattern recognition of the positive and negative DM's answers in regarding to his preferences expressed as comparisons between lotteries in the gambling approach. The utility evaluation is human-computer dialog between a decision maker and computer-based evaluation tool. It concerns mathematically machine learning, since its basis is the axiomatic approach to decision making theory and stochastic approximation. The latter presents unique stochastic recurrent procedures suitable for computer programming.

The American psychologists Griffiths and Tenenbaum (2006) by analyzing intuitive evaluations in the conditions of repetitive life situations have proved the statistical optimality of human assessment. The major idea is that the new data is interpreted in the framework of a probability model built in their consciousness. That means that the Bayesian approach is a natural basis on which human beings formed their decisions, using their previous empirical experience. In such case the *utility theory* and its prescription to make decision based on value (utility) model as mathematical representation of the preferences has another scientific validation of the axiomatic approach in *decision making*. This modeling and its implementation in design is one of the directions of *value driven design*. The latter is a system engineering strategy, which enables multidisciplinary

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design optimization by providing designers with an objective function.

People's preferences contain uncertainty of probabilistic nature due to qualitative type both of the empirical expert information and human notions. A possible approach for solution of these problems is *stochastic programming* (Aizerman et al., 1970). The uncertainty of the subjective preferences could be taken as an additive noise that could be eliminated, as is typical in the stochastic approximation procedures and *machine-learning* based on the stochastic programming.

The objective of the article is to present an innovative approach to value driven modeling of management (control) that bases on preference-oriented decision making. It is described a decision technology that realizes measurement (value-based evaluation) of human's objective-oriented preferences as analytic utility function. The latter is used in human-centered modeling of management/control that bases on the value-oriented determination of requirements and preferences of a human being in representation of complex processes. The utility theory and stochastic approximation are possible solution of this problem that results in a value-based approach to modeling of complex systems. It is demonstrated the application of the described value driven approach to management modeling in design of portfolio optimal control.

BACKGROUND

In complex processes and situations, there is a lack of measurements or even clearly identifiable scales for the basic heuristic information. Internal human expectations and heuristic are generally expressed by qualitative preferences. That is why the common sources of information in such a basic level are the human preferences. Probability theory, stochastic programming and expected utility theory address decision making under these conditions (von Neumann & Morgenstern, 1953; Raiffa, 1968; Pfanzagl, 1970; Fishburn, 1970). The mathematical description on such a fundamental level requires basic mathematical terms like sets, relations and operations over them. In this aspect of mathematical descriptions it is entered the theory of measurements and scaling and utility theory.

From practical point of view the empirical system of human preferences relations is an algebraic system with relations. *System with relations* (SR) is called the set $\boldsymbol{A}$ in conjunction with a set of relations $\boldsymbol{R_i}$, $\boldsymbol{i}\in\boldsymbol{I}$, $\boldsymbol{I}=\{1,2,3,...,n\}$ defined over it and denoted by ($\boldsymbol{A}$, ($\boldsymbol{R_i}$), $\boldsymbol{i}\in\boldsymbol{I}$). In this manner it is introduced a structure in the set $\boldsymbol{A}$ (Pfanzagl, 1970). Relation of *congruency* is called a relation of equivalency ($\approx$) (reflexive, symmetric and transitive) defined over the basic set $\boldsymbol{A}$, if the property of *substitution* is satisfied, i.e. from the fulfillment of relations $(x_1, x_2, x_3,....,x_{hi})\in\boldsymbol{A}^{hi}$ and $(x_j\approx y_j)$ for every j=1, 2, 3 4,...,h_i it follows that $\boldsymbol{R_i}(x_1, x_2, x_3,....,x_{hi})=\boldsymbol{R_i}(y_1, y_2, y_3,....,y_{hi})$for $\forall\boldsymbol{i}$, $\boldsymbol{i}\in\boldsymbol{I}$. By definition the relation of equivalency ($\approx_2$) is coarser than the equivalency ($\approx_1$), if the inclusion $(\approx_1) \subseteq (\approx_2)$ is satisfied. It is known that there exists the coarsest relation ($\approx_A$) over SR ($\boldsymbol{A}$, ($\boldsymbol{R_i}$), $\boldsymbol{i}\in\boldsymbol{I}$). This means that if two elements are in congruency relation ($x\approx_A y$), then they are undistinguishable, with respect to the properties, in the set $\boldsymbol{A}$, described with the set of relations (($\boldsymbol{R_i}$), $\boldsymbol{i}\in\boldsymbol{I}$). If the set $\boldsymbol{A}$ is factorized by the coarsest congruency ($\approx_A$), then in the factor set $\boldsymbol{A}/\approx_A$ the congruency ($\approx_A$) is in fact equality (=). A SR ($\boldsymbol{A}$, ($\boldsymbol{R_i}$), $\boldsymbol{i}\in\boldsymbol{I}$), in which the congruency ($\approx_A$) is the coarsest is called *irreducible*. In this case SR ($\boldsymbol{A}/\approx_A$,($\boldsymbol{R_i}$), $\boldsymbol{i}\in\boldsymbol{I}$) is irreducible.

A homomorphism is an image f, f: $\boldsymbol{A}\rightarrow\boldsymbol{B}$ between two SR ($\boldsymbol{A}$,($\boldsymbol{R_i}$), $\boldsymbol{i}\in\boldsymbol{I}$) and ($\boldsymbol{B}$, ($\boldsymbol{S_i}$), $\boldsymbol{i}\in\boldsymbol{I}$) from the same type, for which $\forall\boldsymbol{i}$, $\boldsymbol{i}\in\boldsymbol{I}$ and $(x_1, x_2, x_3,....,x_{hi})\in\boldsymbol{R_i}$ is satisfied $\boldsymbol{R_i}\,(x_1, x_2, x_3,....,x_{hi}) \Leftrightarrow \boldsymbol{S_i}\,(f(x_1), f(x_2), f(x_3),...., f(x_{hi}))$.

DEFINITION: Every homomorphism from irreducible empirical system into the number system SR ($\boldsymbol{A}$, ($\boldsymbol{Q_i}$), $\boldsymbol{i}\in\boldsymbol{I}$) is called k-dimensional scale.

The empirical system of relations SR is an object from the reality with the properties described by the relations in SR, while the numbered system

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