Latest Advances on Benders Decomposition

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INTRODUCTION

Nowadays, the increased demand for energy and goods around the world has made the business environment more and more complex concerning the design and operation of power systems, supply chain and transportation networks, production scheduling of factories etc. This complexity has made real case studies increasingly difficult to solve due to the large-scale mathematical models constructed. These models do not only depend on the number of parameters, decision variables and constraints. Frequently, even if these features are moderate, optimization problems could still be considered as large-scale due to complicating structures making their solution hard to get.

Information Science and Technology together with Operations Research and Mathematical Programming can show the way towards better solutions in all fields of economy. Nowadays, the evolution of computers combined with mathematical decomposition techniques can solve real size mathematical models and optimize the operation of real-life systems by e.g. avoiding an unpleasant outcome, decreasing operations cost or increasing the overall profit.

The mathematical decomposition techniques aim at decomposing the initial large-scale mathematical model into two or more sub-models based on mathematical programming theorems, as for example, the duality theorem. In principle, the mathematical decomposition techniques do not depend on the case study, but on the structure of the developed mathematical model, which describes the system under study. Mathematical decomposition techniques are:

- Benders decomposition (Benders, 1962),
- Dantzig-Wolfe decomposition (Dantzig & Wolfe, 1960),
- Lagrangian relaxation,
- Lagrangian decomposition and
- Cross decomposition.

The main goal of this chapter is to present the Benders decomposition method, identify its weaknesses and display all the recent research made to tackle them.

In the next section, a very brief literature review on the method is provided. Following, the method is presented and explained together with its main weaknesses. The same section includes the latest advances from the literature, which address these weaknesses. The next section, future and emerging trends on Benders decomposition are discussed. Finally, the chapter is concluded at the final section.

BACKGROUND

Benders decomposition technique was introduced by (Benders, 1962) for linear problems with coupling integer variables. Although the algorithm was initially used for block-decomposed large-scale optimization problem, lately the algorithm

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Table 1. Block-decomposable structure of a problem suitable for Benders Decomposition approach

	a_{11}	a_{12}	a_{13}	…	0	0	0	…	a_{1j-2}	a_{1j-1}	a_{1j}
	a_{21}	a_{22}	a_{23}	…	0	0	0	…	a_{2j-2}	a_{2j-1}	a_{2j}
	…	…	…	…	…	…	…	…	…	…	…
	0	0	0	…	a_{nk-1}	a_{nk}	a_{nk+1}	…	a_{nj-2}	a_{nj-1}	a_{nj}
A =	0	0	0	…	a_{n+1k-1}	a_{n+1k}	a_{n+1k+1}	…	a_{n+1j-2}	a_{n+1j-1}	a_{n+2j}
	…	…	…	…	…	…	…	…	…	…	…
	0	0	0	…	0	0	0	…	a_{i-1j-2}	a_{i-1j-1}	a_{i-1j}
	0	0	0	…	0	0	0	…	a_{ij-2}	a_{ij-1}	a_{ij}

has been applied also in other types of problems. The Benders method has been applied successfully to stochastic programming (Watkins, McKinney, Lasdon, Nielsen, & Martin, 2000) for the solution of multi-scenario, two-stage and multistage stochastic problems where the scenarios are split to sub-scenarios and studied separately decreasing the amount of data taken under consideration in each optimization step; to global optimization problems (Zhu & Kuno, 2003) to address the non-convexity nature of big data; and more recently to build mathematical models (Ierapetriou & Saharidis, 2009), where a large amount of experimental data are available for model building.

BENDERS DECOMPOSITION APPROACH

Presentation of the method

The Benders decomposition (Benders, 1962) technique is based on the idea of exploiting decomposable structure of a given problem so that its solution can be converted into the solution of several smaller sub-problems. *Table 1* shows this block-decomposable structure, where *A* is the matrix of the constraints coefficients.

For the construction of the subproblems, certain variables of the original problem are considered to be complicating. By fixing them, the original problem is decomposed into the Relaxed Master Problem (RMP), which contains only the complicating variables, and the Primal Sub-Problem (PSP), which contains the rest of the variables. Thus, RMP is a relaxation of the original problem and is expected to provide the optimal solution after the addition of a number of constraints (i.e. inequalities).

Without loss of generality, one could consider the following linear problem:

Original Problem (OP):

$$
\begin{aligned}
&Min\ c^T x + d^T y \\
&st. \\
&Ax + By \leq b \\
&x \in \Re_+^n, y \in Z_+^q
\end{aligned}
$$

where $c \in \Re^n, d \in \Re^q, b \in \Re^m$, *A* and *B* are mxn and mxq matrices respectively.

Assuming vector *y* contains a number of decision variables, which are considered to be complicating, the decision variables are partitioned into two sets *x* and *y* and the OP decomposes into the following problems: