Design of Compensators for Comb Decimation Filters

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INTRODUCTION

Decimation is the process of decreasing the sampling rate by an integer, called decimation factor. Decimation has applications in communications, audio signal processing, Sigma Delta Analog to Digital converters, among others. In order to prevent aliasing (unwanted replicas of the input signal), the signal must be previously filtered by a low pass filter, called decimation filter (Jovanovic Dolecek, 2003). The comb filter is the simplest decimation filter usually used in the first decimation stage (Hogenauer, 1981). This filter does not require multipliers, because all its coefficients are equal to unity. In order to achieve correct performance, the comb decimation filter should have a flat pass band of interest. However, comb magnitude characteristic has a droop in the pass band of interest which may deteriorate the decimated signal. The solution is to compensate for a comb pass band droop by an additional simple filter, called compensator. Different methods are proposed for compensator designs. The objective of this paper is to categorize and describe the most important methods, proposed so far, and to propose some future direction for the compensator designs.

BACKGROUND

The transfer function of comb filter is given by the following equation:

$$H(z)=\left[\frac{1}{M}\frac{1-z^{-M}}{1-z^{-}}\right]^{K} \tag{1}$$

where M is the decimation factor and K is the order of the filter.

The magnitude response of the filter is given as:

$$\left|H(e^{j\omega})\right|=\left|\frac{1}{M}\frac{\sin(\omega M/2)}{\sin(\omega/2)}\right|^{K} \tag{2}$$

The comb pass band is defined by the pass band edge (Kwentus &Willson, 1997):

$$\omega_p=\pi/RM \tag{3}$$

where R is the decimation factor of the stage that follows the comb decimation stage.

For values $R<4$, the pass band is considered as a wideband, and in an opposite case it is a narrowband.

As an example, Figure 1 shows the wide pass band zoom ($R=2$), of the magnitude response of comb filter with the decimation factor $M=12$ and an order equal to $K=3$. Note that the response is not flat and has a droop, which increases with the increase of the frequency ω. The inverse comb magnitude characteristic:

$$\left|H_i(e^{j\omega})\right|=\left|\frac{1}{H(e^{j\omega})}\right|=\left|\frac{M\sin(\omega/2)}{\sin(\omega M/2)}\right|^{K} \tag{4}$$

is also shown.

The product of the magnitude characteristics (2) and (4) results in unity:

$$\left|H(e^{j\omega})\right|\left|H_i(e^{j\omega})\right|=1 \tag{5}$$

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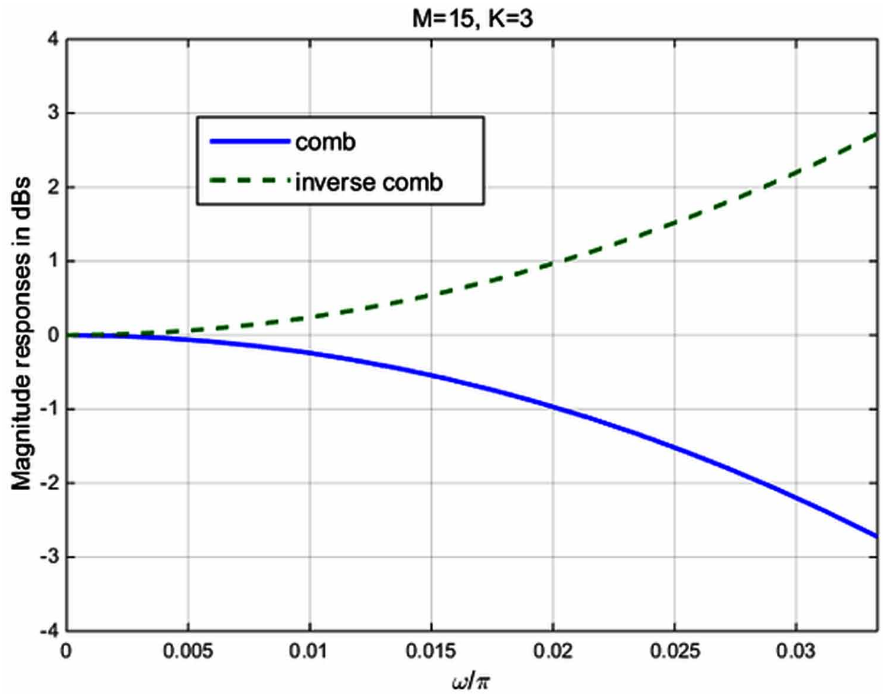
Figure 1. Magnitude and inverse magnitude characteristics of comb, M=15, K=3, R=2



Consequently, in order to get a flat comb magnitude characteristic it is necessary to cascade comb with a filter which has magnitude characteristic approximately equal to the inverse comb magnitude characteristic in the pass band. This filter is called a compensation filter. Denoting the magnitude characteristic of compensator as $\left|G(e^{j\omega})\right|$ it follows:

$$\left|G(e^{j\omega})\right| \approx \left|H_i(e^{j\omega})\right|, \text{ for } 0 \leq \omega \leq \omega_p \qquad (6)$$

where ω_p is the pass band edge defined in (3).

Usually, compensation filter works at a low rate, i.e. after decimation. As a consequence, at high input rate, compensator is expanded by *M*.

The compensated comb is the cascade of comb and compensator. The corresponding transfer function at high input rate is:

$$H_{comp}(z) = H(z)G(z^M) \qquad (7)$$

There are two principal reasons why an inverse comb filter cannot be a compensator: 1) Comb filter has zeros on the unit circle, becoming poles in the inverse comb filter, thus resulting in instability. 2) The resulting magnitude characteristic is equal unity for all frequencies. However, the unity magnitude characteristic is only required for the pass band.

Consequently, compensator should be designed to approximate inverse comb magnitude characteristic only in the pass band, and to no deteriorate comb stop band characteristic. Additionally, knowing that a comb filter is a very simple multiplier less filter, its compensator should be also a simple and desirable multiplier less filter.

REVIEW OF METHODS FOR DESIGN OF COMB COMPENSATORS

Different methods for design of comb compensators are advanced in the literature. The com-