Chapter 47
The Concept of the Shapley Value and the Cost Allocation Between Cooperating Participants

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ABSTRACT

The goal of this chapter is to illustrate two mathematical game theory concepts for allocating costs (savings) between cooperating participants, specifically in healthcare settings. These concepts are the nucleolus and the Shapley value. The focus of this chapter is on the practical application of the Shapley value for the cost sharing within the bundled payments model for the episodes of care mandated recently by the Center for Medicare Services (CMS). The general Shapley value methodology is illustrated, as well as an important particular case in which each participant uses only a portion of the largest participant's asset (the so-called airport game). The intended readers are primarily leaders of organizations and hospitals involved in the implementation of the CMS-mandated bundled payment model for the episodes of care.

INTRODUCTION

Game theory is a branch of applied mathematics that studies strategic situations in which participants (players) act in order to maximize their returns (payoffs). As such, game theory provides models of rational behavior (decision-making) for strategic interactions.

Many types of problems that involve decision strategies for cooperating or non-cooperating participants present a fruitful ground for the application of mathematical game theory (Dowd, 2004; Cachon & Netessine, 2004).

In particular, cost allocation problems arise in many situations in which participants work together, such as healthcare providers who have to coordinate patient care in order to reduce the cost and improve the quality of care. It is demonstrated that a natural framework for developing a methodology for cost

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allocation problems could be based on game theoretical concepts (Tijs and Driessen, 1986; Roth, 1988; Young, 1994; Moulin, 2003). Several concepts for determining the 'fair' cost allocation have been proposed but only a few of these concepts have been used in practice: the nucleolus and the Shapley value.

In this chapter, these two concepts are illustrated side by side. The focus is on examples of practical application of the Shapley value, specifically in healthcare settings. The following two cases are considered in details: (i) the general application of the Shapley value methodology for cost allocation between cooperating providers of care applied to the bundled payment model mandated recently by the Center for Medicare Services' (CMS), and (ii) an important particular case, in which each participant uses only a portion of the largest participant's asset (the so-called airport game).

BACKGROUND

By pooling resources and cooperating the participants usually reduce the total joint costs and realize savings. The question arises is how the reduced costs or the realized savings should be fairly allocated between them.

There could be different definitions of fair allocation. Some of them are:

- **Equitable Allocation:** Gives everyone the same satisfaction level, i.e. the proportion each player receives by their own valuation is the same for all of them. This is a difficult aim as players might not be truthful if asked their valuation.
- **Proportional Allocation:** Guarantees that each player gets his share. For instance, if three people divide up an asset then each gets at least a third by their own valuation.
- **Envy-Free Allocation:** Everyone prefers his own share to the others. No one is jealous of anyone else. No one would trade his share with anyone else's.
- **An Efficient or Pareto Optimal Allocation:** Ensures that no other allocation would make someone better off without making someone else worse off. The term efficiency comes from the economics idea of the efficient market.
- **Merit-Based Allocation:** The more one brings to the coalition, the more one gets out of the division of the accumulated gains.

A concept of fairness is rather subjective. It depends on the participants' socio-economic views and other factors. The fairness schemes described in the next sections form a basis of the two most popular cost allocation approaches: the nucleolus (Tijs and Driessen, 1986; Saad, 2009) and the Shapley value (Roth, 1988; Young, 1994).

MAIN FOCUS

The Nucleolus Concept

The nucleolus can be defined as an equilibrium that finds the 'center of gravity' of the so-called core. The core is defined as a set of inequalities that meet the requirement that no participant or a group of