

# Chapter 7

## L(h,k)–Labeling of Intersection Graphs

**Sk. Amanathulla**

*Vidyasagar University, India*

**Madhumangal Pal**

*Vidyasagar University, India*

### ABSTRACT

*One important problem in graph theory is graph coloring or graph labeling. Labeling problem is a well-studied problem due to its wide applications, especially in frequency assignment in (mobile) communication system, coding theory, ray crystallography, radar, circuit design, etc. For two non-negative integers, labeling of a graph is a function from the node set to the set of non-negative integers such that if and if, where it represents the distance between the nodes. Intersection graph is a very important subclass of graph. Unit disc graph, chordal graph, interval graph, circular-arc graph, permutation graph, trapezoid graph, etc. are the important subclasses of intersection graphs. In this chapter, the authors discuss labeling for intersection graphs, specially for interval graphs, circular-arc graphs, permutation graphs, trapezoid graphs, etc., and have presented a lot of results for this problem.*

### INTRODUCTION

Almost all problems in the world can be solve by designing graphs. So, during long period graph theory is being researched. In engineering, physical science, mathematical science, *graph* has lot of applications. One important problem in graph theory is graph coloring or graph labeling.  $L(h,k)$ -labeling problem is a well studied problem due to its wide applications, specially in frequency assignment in (mobile) communication system, coding theory, X-ray crystallography, radar, circuit design, etc. For two non-negative integers  $h$  and  $k$ , an  $L(h,k)$ -labeling of a graph  $G = (V, E)$  is a function  $f$  from the node set  $V$  to the set of non-negative integers such that  $|f(x) - f(y)| \geq h$  if  $d(x, y) = 1$  and  $|f(x) - f(y)| \geq k$  if  $d(x, y) = 2$ , where  $d(x, y)$  represents the distance between the nodes  $x$  and  $y$ .

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Intersection graph is a very important subclasses of graph. Unit disc graph, chordal graph, interval graph, circular-arc graph, permutation graph, trapezoid graph etc. are the important subclasses of intersection graphs.

In this chapter, we discuss  $L(h,k)$ -labeling for intersection graphs, specially, for interval graphs, circular-arc graphs, permutation graphs, trapezoid graphs etc. and have presented a lot of results for this problem.

## **BASIC CONCEPT OF $L(h,k)$ -LABELING**

In this section, the definition and span of  $L(h,k)$ -labeling is presented. Different variations of  $L(h,k)$ -labeling is also highlighted in this section. The definition of  $L(h,k)$ -labeling is as follows.

**Definition 1  $L(h,k)$ -labeling:** Given a graph  $G = (V, E)$  and two nonnegative integers  $h$  and  $k$ , an  $L(h,k)$ -labeling is an assignment of non-negative integers to the nodes of  $G$  such that adjacent nodes are labelled using colours at least  $h$  apart, and nodes having a common neighbour are labelled using colours at least  $k$  apart. The difference between largest and smallest labels is called the span. The aim of the  $L(h,k)$ -labeling problem is to minimize the span. The minimum span over all possible labeling functions is denoted by  $\lambda_{h,k}(G)$  and is called  $\lambda_{h,k}$ -number of  $G$ .

In other words, if  $f(x)$  is the label assigned to the node  $x$  then

$$|f(x) - f(y)| \geq h \text{ if } d(x, y) = 1$$

and

$$|f(x) - f(y)| \geq k \text{ if } d(x, y) = 2,$$

where  $d(x, y)$  is the distance (i.e. number of edges) between  $x$  and  $y$ .

The  $L(h,k)$ -labeling problem can also be referred to as:

- Distance-2-coloring and  $D2$ -node coloring problem (when  $h = k = 1$ );
- Radiocoloring problem and  $\lambda$ -coloring problem (when  $h = 2$  and  $k = 1$ );
- Frequency assignment problem;
- Distance two labeling, etc.

For different values of  $h$  and  $k$  different  $L(h,k)$ -labeling problems are addressed by the researchers, specially  $L(2,1)$ ,  $L(0,1)$  and  $L(1,1)$ -labeling problems. For general graphs, the lower bound for  $\lambda_{2,1}(G)$  is  $\Delta + 1$ . But the upper bound has gradually improved. Griggs and Yeh (1992) proved that  $\lambda_{2,1}(G) \leq \Delta^2 + 2\Delta$  and have proposed the following conjecture.

### **Griggs and Yeh Conjecture**

For a graph  $G$  with maximum degree  $\Delta \geq 2$ ,  $\lambda_{2,1}(G) \leq \Delta^2$ . In 1993, Jonas (1993) has shown that

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