Chapter 7
L(h,k)-Labeling of Intersection Graphs

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ABSTRACT

One important problem in graph theory is graph coloring or graph labeling. Labeling problem is a well-studied problem due to its wide applications, especially in frequency assignment in (mobile) communication system, coding theory, ray crystallography, radar, circuit design, etc. For two non-negative integers, labeling of a graph is a function from the node set to the set of non-negative integers such that if and if, where it represents the distance between the nodes. Intersection graph is a very important subclass of graph. Unit disc graph, chordal graph, interval graph, circular-arc graph, permutation graph, trapezoid graph, etc. are the important subclasses of intersection graphs. In this chapter, the authors discuss labeling for intersection graphs, specially for interval graphs, circular-arc graphs, permutation graphs, trapezoid graphs, etc., and have presented a lot of results for this problem.

INTRODUCTION

Almost all problems in the world can be solve by designing graphs. So, during long period graph theory is being researched. In engineering, physical science, mathematical science, *graph* has lot of applications. One important problem in graph theory is graph coloring or graph labeling. *L*(*h,k*)-labeling problem is a well studied problem due to its wide applications, specially in frequency assignment in (mobile) communication system, coding theory, X-ray crystallography, radar, circuit design, etc. For two non-negative integers h and k, an *L*(*h,k*)-labeling of a graph $G=(V,E)$ is a function f from the node set V to the set of non-negative integers such that $|f(x)-f(y)|\geq h$ if $d(x,y)=1$ and $|f(x)-f(y)|\geq k$ if $d(x,y)=2$, where $d(x,y)$ represents the distance between the nodes x and y.

DOI: 10.4018/978-1-5225-9380-5.ch007

Intersection graph is a very important subclasses of graph. Unit disc graph, chordal graph, interval graph, circular-arc graph, permutation graph, trapezoid graph etc. are the important subclasses of intersection graphs.

In this chapter, we discuss *L*(*h,k*)-labeling for intersection graphs, specially, for interval graphs, circular-arc graphs, permutation graphs, trapezoid graphs etc. and have presented a lot of results for this problem.

BASIC CONCEPT OF *L*(*h,k*)-LABELING

In this section, the definition and span of *L*(*h,k*)-labeling is presented. Different variations of *L*(*h,k*)-labeling is also highlighted in this section. The definition of *L*(*h,k*)-labeling is as follows.

Definition 1 *L*(*h,k*)-labeling: Given a graph $G=(V,E)$ and two nonnegative integers h and k, an *L*(*h,k*)-labeling is an assignment of non-negative integers to the nodes of G such that adjacent nodes are labelled using colours at least h apart, and nodes having a common neighbour are labelled using colours at least k apart. The difference between largest and smallest labels is called the span. The aim of the $L(h,k)$-labeling problem is to minimize the span. The minimum span over all possible labeling functions is denoted by $\lambda_{h,k}(G)$ and is called $\lambda_{h,k}$-number of G.

In other words, if $f(x)$ is the label assigned to the node x then

$$| f(x)-f(y) | \geq h \text{ if } d(x,y)=1$$

and

$$| f(x)-f(y) | \geq k \text{ if } d(x,y)=2,$$

where $d(x,y)$ is the distance (i.e. number of edges) between x and y.

The *L*(*h,k*)-labeling problem can also be referred to as:

- Distance-2-coloring and $D2$-node coloring problem (when $h=k=1$);
- Radiocoloring problem and λ-coloring problem (when $h=2$ and $k=1$);
- Frequency assignment problem;
- Distance two labeling, etc.

For different values of h and k different *L*(*h,k*)-labeling problems are addressed by the researchers, specially $L(2,1)$, $L(0,1)$ and $L(1,1)$-labeling problems. For general graphs, the lower bound for $\lambda_{2,1}(G)$ is $\Delta+1$. But the upper bound has gradually improved. Griggs and Yeh (1992) proved that $\lambda_{2,1}(G) \leq \Delta^2+2\Delta$ and have proposed the following conjecture.

Griggs and Yeh Conjecture

For a graph G with maximum degree $\Delta \geq 2$, $\lambda_{2,1}(G) \leq \Delta^2$. In 1993, Jonas (1993) has shown that