Chapter 19
Fuzzy Graphs and Fuzzy Hypergraphs

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ABSTRACT

Relationship is the core building block of a network, and today's world advances through the complex networks. Graph theory deals with such problems more efficiently. But whenever vagueness or imprecision arises in such relationships, fuzzy graph theory helps. However, fuzzy hypergraphs are more advanced generalization of fuzzy graphs. Whenever there is a need to define multiary relationship rather than binary relationship, one can use fuzzy hypergraphs. In this chapter, interval-valued fuzzy hypergraph is discussed which is a generalization of fuzzy hypergraph. Several approaches to find shortest path between two given nodes in an interval-valued fuzzy graphs is described here. Many researchers have focused on fuzzy shortest path problem in a network due to its importance to many applications such as communications, routing, transportation, etc.

INTRODUCTION

Graph is a representation of relationship between objects. It can give a good idea of scope of relationship for any complex networking model. Graph has many variations such as directed graph, undirected graph, simple graph, pseudo graph, multi graph, finite graph, infinite graph, etc. In a directed graph the relation defined on V is not symmetric but in undirected graph the relation defined on V is symmetric. In a graph, loops may occur, that is, a vertex may have a relation to itself. Also, there may have more than one edges between two vertices, called parallel edges. Simple graphs have no multiple edges and loops at all. If in a graph, there are finite number of vertices and finite number of edges, then it is called finite graph otherwise, it is infinite graph. Most commonly, unless stated otherwise, graph means undirected

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simple finite graph. In general, any mathematical problem involving points and connections among them can be called a graph and its pictorial representation may lead to a solution. Thus, graph as mathematical model of some problem can solve a graph-theoretic problem and then presents the solution of original problem.

Fuzzy graph is rather extension of (crisp) graph by introducing the concepts of fuzzy sets and fuzzy relations instead of (crisp) sets. The notion of fuzzy sets and fuzzy relations were first introduced by Rosenfeld (1975). After Rosenfeld, in 1977, Kaufmann (1973) introduced the notion of fuzzy hypergraphs.

FUZZY GRAPH

A **fuzzy graph** $\xi = \left(V, \sigma, \mu\right)$ is a triplet consisting of a non-empty set V together with a pair of functions $\sigma : V \to \left[0,1\right]$ and $\mu : V \times V \to \left[0,1\right]$ such that

$$\mu\left(xy\right) \le min\left\{\sigma\left(x\right), \sigma\left(y\right)\right\}$$

for all $x, y \in V$.

Here the two fuzzy sets σ and μ are called **fuzzy vertex set** and **fuzzy edge set** of ξ respectively. Clearly, μ is a fuzzy relation on σ.

The fuzzy subgraph of a fuzzy graph is a fuzzy graph whose fuzzy set is a subset of the fuzzy set of the given fuzzy graph.

A fuzzy graph $\xi = \left(V', \tau, \nu\right)$ is said to be a **partial fuzzy subgraph** of ξ if $\tau \subseteq \sigma$ and $\nu \subseteq \mu$.

The fuzzy graph $\xi = \left(V', \tau, \nu\right)$ is called a **fuzzy subgraph** of ξ if $\tau\left(x\right) \le \sigma\left(x\right)$ for all $x \in V'$ and $\nu\left(x, y\right) \le \mu\left(x, y\right)$ for all $x, y \in V'$ where $V' \subset V$.

A partial fuzzy subgraph $\xi' = \left(V', \tau, \nu\right)$ of ξ is said to span ξ if $\sigma = \tau$. This partial fuzzy subgraph ξ is called a spanning fuzzy subgraph of ξ.

An **underlying crisp graph** of a fuzzy graph $\xi = \left(V, \sigma, \mu\right)$ is a crisp graph $\xi' = \left(V, \sigma', \mu'\right)$ where

$$\sigma' = \left\{u \in V\left(\xi\right) \middle| \sigma\left(u\right) \rangle 0\right\}$$

and

$$\mu' = \left\{\left(u, v\right) \middle| \mu\left(u, v\right) \rangle 0\right\}.$$

A **path** P in a fuzzy graph $\xi = \left(V, \sigma, \mu\right)$ is a sequence of distinct vertices $v_1, v_2, \ldots, v_n \left(n \ge 2\right)$ such that

$$\mu\left(v_i, v_{i+1}\right) > 0, i = 1, 2, \ldots, \left(n-1\right).$$