

Chapter 9

Stability Analysis of a Nonlinear Epidemic Model With Generalized Piecewise Constant Argument

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ABSTRACT

The authors consider a nonlinear epidemic equation by modeling it with generalized piecewise constant argument (GPCA). The authors investigate invariance region for the considered model. Sufficient conditions guaranteeing the existence and uniqueness of the solutions of the model are given by creating integral equations. An important auxiliary result giving a relation between the values of the unknown function solutions at the deviation argument and at any time t is indicated. By using Lyapunov-Razumikhin method developed by Akhmet and Aruğaslan for the differential equations with generalized piecewise constant argument (EPCAG), the stability of the trivial equilibrium is investigated in addition to the stability examination of the positive equilibrium transformed into the trivial equilibrium. Then sufficient conditions for the uniform stability and the uniform asymptotic stability of trivial equilibrium and the positive equilibrium are given.

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INTRODUCTION

In this chapter, the authors take a differential equation in population dynamics mentioned by Huang, Liu, and Fory's (2016) into account. The considered equation represents a generalization of the SIS model established by Cooke (1979). The authors develop this model with a GPCA since change between the GPCA can be arbitrarily chosen. Since the taken model has a nonlinear function with GPCA, the model and the examinations performed for it are remarkable. As the beginning of these examinations, invariance region for the considered equation with GPCA is investigated by the authors. Besides, the authors give sufficient conditions for the existence and uniqueness of the trivial equilibrium and the positive equilibrium of the proposed model. Next, the authors give sufficient conditions guaranteeing the existence and uniqueness of the solutions of the nonlinear epidemic model by creating integral equations. The authors indicate an important auxiliary result giving a relation between the values of the unknown function at the deviation argument and at any time t of the proposed model. By using Lyapunov-Razumikhin method developed by Akhmet and Aruğaslan (2009) for EPCAG, the authors investigate the stability of the trivial equilibrium for the considered nonlinear epidemic model with GPCA. Moreover, based on the theoretical results in the paper (Akhmet & Aruğaslan, 2009), the authors investigate the stability of the positive equilibrium point by transforming it into the trivial equilibrium. Then, sufficient conditions for uniform stability and uniform asymptotic stability of the trivial equilibrium and the transformed positive equilibrium are given. Thus, the authors have reached the results depending on the parameters of the considered equation. During all these investigations, the nature of the solutions is evaluated within the biologically meaningful range $[0,1]$ as required by the examination performed for the positive invariance region.

BACKGROUND

Differential equations are very valuable in understanding the real life problems since they allow the mathematical expression of the real phenomena. However, modeling of problems with ordinary differential equations is often not enough. Because, while problems are set up mathematically by neglecting the discontinuous effects, the models and thus the results of their qualitative analysis are far distance from the reality. This necessitated the introduction and development of the theory of differential equations with discontinuities. One type of equation developed as a result of this requirement is the differential equations with deviating arguments. Differential equations with deviating arguments host many classes of equations, such as functional differential equations, differential equations with delay, piecewise

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