Chapter 10
The Parseval Equality and Expansion Formula for Singular Hahn-Dirac System

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ABSTRACT

This work studies the singular Hahn-Dirac system given by

$$\begin{pmatrix} 0 & -\frac{1}{q}D_{-\omega q^{-1},q^{-1}} \\ D_{\omega,q} & 0 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}+\begin{pmatrix} p(x) & 0 \\ 0 & r(x) \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}=\mu\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}.$$

Here μ is a complex spectral parameter, $p(.)$ and $r(.)$ are real-valued continuous functions at $\omega 0$, defined on $[\omega 0,\infty)$ and $q\in(0,1)$, $\omega_0 := \omega/(1-q)$, $\omega>0$, $x\in[\omega 0,\infty)$. The existence of a spectral function for this system is proved. Further, a Parseval equality and an expansion formula in eigenfunctions are proved in terms of the spectral function.

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INTRODUCTION

The theory of Hahn difference operator was introduced by Hahn in 1949; see the papers (Hahn, 1949,1983). This operator provides a unifying structure for the study of the forward difference operator defined by

$$\Delta_{\omega} f(x) := \frac{f(\omega + x) - f(x)}{(\omega + x) - x}, \; x \in \mathbb{R}$$

and the study of the quantum q-difference operator (Jackson, 1910) defined by

$$D_q f(x) := \frac{f(qx) - f(x)}{qx - x}, \; x \neq 0.$$

Recently, Hahn difference operators are receiving an increase of interest due to their applications in the construction of families of orthogonal polynomials and approximation problems (see Alvarez-Nodarse, 2006; Dobrogowsa & Odzijewicz, 2006; Kwon, Lee, Park & Yoo, 1998; Lesky, 2005; Petronilho, 2007 and the references therein).

In the literature there exist some papers studying the Hahn difference operator. The theory of linear Hahn difference equations was developed in the paper (Hamza & Ahmed, 2013). In (Hamza & Ahmed, 2013), the authors also study the existence and uniqueness of the solution for initial value problems related to Hahn difference equations. Hamza and Makharesh (Hamza & Makheresh, 2016) investigated Leibniz's rule and Fubini's theorem associated with the Hahn difference operator. The nonlocal boundary value problem for nonlinear Hahn difference equation was developed in the paper (Sitthiwirattham, 2016). In 2018, (Annaby, Hamza & Makherseh, 2018), the regular Hahn-Sturm-Liouville problem

$$-\frac{1}{q} D_{-\omega q^{-1}, q^{-1}} D_{\omega, q} y(x) + v(x) y(x) = \lambda y(x),$$

$$a_1 y(\omega_0) + a_2 D_{-\omega q^{-1}, q^{-1}} y(\omega_0) = 0,$$

$$b_1 y(b) + b_2 D_{-\omega q^{-1}, q^{-1}} y(b) = 0,$$