Chapter 11
The Stability of an Epidemic Model With Piecewise Constant Argument by Lyapunov-Razumikhin Method

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ABSTRACT

The authors propose a nonlinear epidemic model by developing it with generalized piecewise constant argument (GPCA) introduced by Akhmet. The authors investigate invariance region for the considered model. For the taken model into consideration, they obtain a useful inequality concerning relation between the values of the solutions at the deviation argument and at any time for the epidemic model. The authors reach sufficient conditions for the existence and uniqueness of the solutions. Then, based on Lyapunov-Razumikhin method developed by Akhmet and Aruğaslan for the differential equations with generalized piecewise constant argument (EPCAG), sufficient conditions for the stability of the trivial equilibrium and the positive equilibrium are investigated. Thus, the theoretical results concerning the uniform stability of the equilibriums are given.

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INTRODUCTION

In the present chapter, the main objective is to provide information on the stability analysis of the susceptible-infected-susceptible (SIS) model established by Cooke (1979). This model is developed by GPCA which is defined by Akhmet (2007a, 2007b). Firstly, invariance region for the considered equations with GPCA is investigated by the authors, respectively. The inequalities describing the relationship between the values received at this piecewise function and at any given time for the solutions of the models are given. This inequality is useful and important in the proofs of stability analysis in the sense of Lyapunov-Razumikhin method. It is aimed to obtain sufficient conditions guaranteeing the existence and uniqueness of the solutions of the proposed model. Afterwards, it is aimed to perform stability analysis with the help of Lyapunov-Razumikhin method developed by Akhmet and Aruğaslan (2009) (Aruğaslan, 2009) for EPCAG. Based on the relevant method, the conditions that guarantee the uniform stability and the uniform asymptotic stability of the trivial equilibrium and the positive equilibrium are presented. The obtained theoretical results depend on the parameters of the equation.

BACKGROUND

The mathematical evaluation of the problems encountered in real life and the interpretation of their past and future dynamics has been and continues to be a subject that attracted the attention of the scientific world. In this direction, it is possible to express these problems as mathematical models by differential equations. With the help of the qualitative theory of differential equations, informations about the behaviors of these models can be provided. However, the fact that these information presents distant results from the reality phenomenon strengthens the likelihood of misleading the behavior of real processes. In this situation, it is possible to reach the findings that will affect the life negatively by the models which do not fully reflect the dynamics of real life problems. In order to overcome this issue, it would be a step in the right direction to consider differential equations with deviation argument instead of ordinary differential equations. Because, differential equations with deviation argument improve models representing real life problems, and moreover they contribute to understanding how a situation in the past of these problems affect the current and subsequent state of them. In other words, the effects of a past value of the models that has been constructed and developed with the help of such equations on the current behavior can be observed. Therefore, the efforts to achieve the development of such equations have received considerable attention by many scientists. As a result of these efforts, the qualitative theory of such equations