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Performance Characteristics of Discrete-Time Queue With Variant Working Vacations

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ABSTRACT

This article analyzes an infinite buffer discrete-time single server queueing system with variant working vacations in which customers arrive according to a geometric process. As soon as the system becomes empty, the server takes working vacations. The server will take a maximum number K of working vacations until either he finds at least on customer in the queue or the server has exhaustively taken all the vacations. The service times during regular busy period, working vacation period and vacation times are assumed to be geometrically distributed. The probability generating function of the steady-state probabilities and the closed form expressions of the system size when the server is in different states have been derived. In addition, some other performance measures, their monotonicity with respect to K and a cost model are presented to determine the optimal service rate during working vacation.

KEYWORDS

Geometric Distribution, Probability Generating Function, Queue, Variant Working Vacations

INTRODUCTION

Unlike the classical vacation queues where the server suspends the services temporarily, in working vacation (WV) queues the server is active during the vacation period which is called working vacation (WV), Servi and Finn (2002). Wu and Takagi (2006) generalized Servi and Finn's (2002) *M/M/1/WV* queue to an *M/G/1/WV* queue. Banik et al. (2007) studied a general input *GI/M/1/N/WV* queue. The stochastic decomposition results of an *M/M/1* queue with WV have been derived by Liu et al. (2007) and the corresponding M/G/1 queue was studied by Li et al. (2009).

The analysis of discrete time queueing models has received considerable attention in view of their application in practical problems that arise from communication and computer systems including time-division multiple access (TDMA) schemes, asynchronous transfer mode (ATM), multiplexers in the broadband integrated services digital network (B-ISDN), management in service system and electronic commerce, etc. Past work on discrete-time queues is found in Meisling (1958), Hunter (1983), Takagi (1993). Tian and Zhang (2002), Alfa (2003). Li and Tian (2007) considered a discrete-time GI/Geo/1 queue with WV and vacation interruption. Li et al. (2007) considered a discrete-time GI/Geo/1 queue with MWV under Early Arrival System (EAS) and Late Arrival System (LAS) schemes. Li and Tian (2008) and Tian et al. (2008) have analyzed a Geo/Geo/1 queue with single working vacation (SWV) and multiple working vacation (MWV), respectively. A discrete-time renewal input finite buffer batch service queue with MWVs has been studied by Vijaya Laxmi and Jyothsna (2014) using the supplementary variable techniqeue and the corresponding queue with balking and SWV has been presented by Vijaya Laxmi et al. (2015). Recently, a retrial queue with working vacation

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for the batch arrival $Geo^X / Geo / 1$ queue has been analyzed by Upadhyaya (2015) under EAS scheme.

In variant working vacation (VWV) policy, unlike the SWV or MWVs, a fixed number of consecutive vacations, say *K*, are taken by the server if the system remains empty at the end of previous vacation termination epoch. This kind of vacation schedule is investigated by Zhang and Tian (2001) for the *Geo/G/1* queue with multiple adaptive vacations. Banik (2009) studied the infinite-buffer single server queue with variant of multiple vacation policy and batch Markovian arrival process by using matrix analytic method. For more literature on this work, see Ke and Chang (2009), Ke et al. (2010) and Wang et al. (2011). Zhang and Hou (2011) analyzed a steady-state *GI/M/1/N* queue with a variant multiple working vacation (VWV) by using matrix analytic method. Yue et al. (2014) analyzed the M/M/1 queueing system with impatient customers and VV and obtained the closed-form expressions of the mean system sizes when the server is in different states using probability generating functions. A finite buffer *M/M/1* queue with VWV, balking and reneging has been analyzed by Vijaya Laxmi and Jyothsna (2014) obtaining the steady state probabilities using matrix form solutions.

The above literature indicates that discrete-time *Geo/Geo/1* queue with VWV has not been studied so far to the best of our knowledge. In digital communication systems, this type of service /vacation policy is important as it reduces the switching and operating cost in case of least loaded systems. Further, the study of VWV queue gives more room for flexible switching from vacation to regular busy period by adjusting the value of *K* as desired by system design requirements.

The primary objectives of the present paper are:

1. to obtain the steady-state probabilities a discrete-time *Geo/Geo/1* queue with VWV under late arrival system with delayed access (LAS-DA) using probability generating functions; and
2. To study the VWV (K-Vacation) policy that covers both MWV $\left(\mathrm{K} \to \infty\right)$ and SWV $\left(\mathrm{K} = 1\right)$.
3. to study performance characteristics and to develop a cost model to determine the optimum service rates that optimize the total expected cost using the quasi-Newton optimization method.

The present model finds applications in many real-life applications like (i) packet switching communication protocols; (ii) the repair garage for automobiles wherein a mechanic repairs the automobiles; (iii) manufacturing and transportation processes, etc.

MODEL DESCRIPTION

Let us consider a discrete-time $Geo / Geo / 1$ queueing system with VWVs under LAS-DA policy. The time is divided into constant length intervals (called slots) and the probability of an arrival and a departure occurring simultaneously is not zero. A potential arrival occurs in the interval $\left(t-,t\right)$ and potential departure occurs in $\left(t,t+\right)$. In order to formulate the model, the following assumptions have been made:

The arrival of a customer to the system occurs at the end of slot $t-$, where $t = 0,1,2,\ldots$. Inter-arrival times (*A*) of two successive arrivals are independent and identically distributed (i.i.d) random variables and follow geometric distribution with probability mass function (*p.m.f*) (Figure 1).

$$P\left\{A = m\right\} = \lambda\overline{\lambda}^{m-1},\ m \geq 1, 0 < \lambda < 1.$$

where for any real number $x \in \left[0,1\right]$, we denote $\overline{x} = 1 - x$.

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