2

Use of Grey Numbers for Evaluating a System's Performance Under Fuzzy Conditions

Michael Voskoglou
Graduate Technological Educational Institute of Western Greece, Greece

INTRODUCTION

In Management a ***system*** is understood to be a set of interacting components forming an integrated whole and working together for achieving a common target. A factory, a hospital or a bank, constitute standard examples of systems, as well as a physical or biological system, an abstract knowledge system, etc. As a multi- perspective domain systems' theory serves as a bridge for an interdisciplinary dialogue between autonomous areas of study (Balley, 1994, Backlund, 2000). The assessment of a system's performance constitutes a very important part of this theory, because it enables the designers of the system to correct its weaknesses and therefore to increase its effectiveness.

When the performance of a system's components is evaluated by numerical scores, then the traditional way for assessing the system's ***mean performance*** is the calculation of the average of those scores. However, in order to comfort the reviewer's existing uncertainty about the exact value of the numerical scores corresponding to each of the system's components, frequently in practice the assessment is made not by numerical scores but by linguistic grades, like excellent, very good, good, etc. This involves a degree of fuzziness and makes the calculation of the mean value of those grades impossible.

A popular in such cases method for evaluating the overall system's performance is the calculation of the ***Grade Point Average (GPA) index*** (e.g. see Voskoglou, 2017b, Chapter 6, p.125). However, GPA is a weighted average in which greater coefficients (weights) are assigned to the higher grades, which means that it reflects not the mean, as we wish, but the ***quality performance*** of the system.

Analysis of the objectives and of the scope of the present work: In order to overcome the above difficulties, we have utilized in earlier works the system's *total uncertainty* under fuzzy conditions (created by the qualitative assessment of its components) as a measure of its effectiveness (Voskoglou, 2017b, Chapter 5). This manipulation is based on a fundamental principle of the Information Theory according to which the reduction of a system's uncertainty is connected to the increase of information obtained by a system's activity. In other words, lower uncertainty indicates a greater amount of information and therefore a better system's performance with respect to the corresponding activity. However, this method needs laborious calculations, cannot give a precise qualitative characterization of a system's performance and, most importantly, it is applicable for comparing the performance of two different systems with respect to a common activity only under the assumption that the existing uncertainty is the same in the two systems before the activity.

For this reason, ***Fuzzy Numbers (FNs)*** have been also used in later works for assessing a system's mean performance under fuzzy conditions (Voskoglou, 2017a). However it was observed that, although the calculation of three components is needed for expressing the mean value of the qualitative grades in the form of a ***Triangular FN (TFN)***, only the middle component is used for defuzzifying it. The above

DOI: 10.4018/978-1-7998-3479-3.ch023

observation suggests the search for a method analogous to the use of TFNs that possibly reduces the required computational burden. This search led us to utilizing ***Grey Numbers (GNs)*** (Liu & Lin, 2010) as an alternative tool for assessing a system's mean performance with qualitative grades (Voskoglou & Theodorou, 2017). Although this new method has been proved to be equivalent with the use of TFNs, it needs the calculation of two components only (instead of three) for expressing the mean value of the qualitative grades in the form of a GN.

Synthesis of the present work: Here we are going to present these two innovative methods and to prove their equivalence. The rest of the Chapter is formulated as follows: In the second (background) section the assessment method with the TFNs is sketched and the necessary information about GNs is given, needed for the understanding of the present work. The alternative assessment method using GNs is developed in the third section and its equivalence is proved to the method with TFNs. Examples illustrating those methods are presented in the fourth section and the perspectives of future research on the subject are discussed in the fifth section. Finally, the sixth section includes the conclusions of the present research.

BACKGROUND

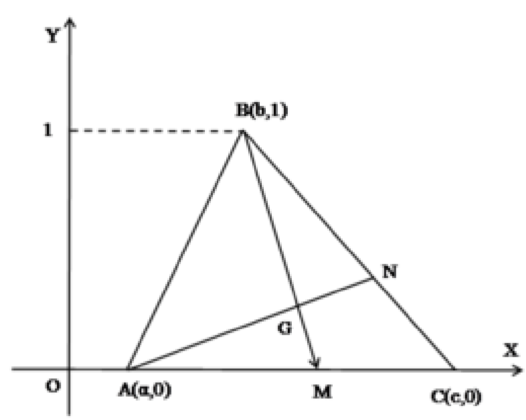
Triangular Fuzzy Numbers (TFNs)

The reader is supposed to be familiar with the basic principles of the theory of ***Fuzzy Sets (FSs)*** and the books of Klir & Folger (1988) and of Dubois & Prade (1980) are proposed as general references on the subject.

A ***Fuzzy Number (FN)***, say A, is a **FS** on the set ***R*** of the real numbers, which is ***normal*** (i.e. there exists x in ***R*** such that $m_A(x) = 1$) and ***convex*** (i.e. all its ***a-cuts*** $A^a = \{x \in U: m_A(x) \geq a\}$, a in [0, 1] are closed real intervals) and whose ***membership function*** $y = m_A(x)$ is a piecewise continuous function. For general facts on FNs the reader may study the classical on the subject book of Kaufmann and Gupta (1994)

A TFN (a, b, c), with a, b, c real numbers such that $a < b < c$ is the simplest form of a FN representing mathematically the fuzzy statement that "the value of b lies in the interval $[a, c]$". The membership function $y = m(x)$ of (a, b, c) is zero outside the interval $[a, c]$, while its graph in $[a, c]$ consists of two straight line segments forming a triangle with the OX axis (Figure 1).

Figure 1. Graph and Centre of Gravity of the TFN (a, b, c)



Related Content