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Intelligent Constructing Exact Tolerance Limits for Prediction of Future Outcomes Under Parametric Uncertainty

Nicholas Nechval
University of Latvia, Latvia

INTRODUCTION

The logical purpose for a statistical tolerance limit (where the coverage value γ is the percentage of the future process outcomes to be captured by the prediction, and the confidence level $(1-\alpha)$ is the proportion of the time we hope to capture that percentageγ) is to predict future outcomes for some production process which is treated as process, say, with stochastic variation of a product lifetime. The applications of tolerance limits (intervals) are varied. They included clinical and industrial applications, including quality control, applications to environmental monitoring, to the assessment of agreement between two methods or devices, and applications in industrial hygiene. For example, such tolerance limits are required, when planning life tests, engineers may need to predict the number of failures that will occur by the end of the test or to predict the amount of time that it will take for a specified number of units to fail. Tolerance limits of the type mentioned above are considered in this article, which presents a new technique for constructing exact statistical (lower and upper) tolerance limits on outcomes (for example, on order statistics) in future samples. Attention is restricted to the extreme-value and two-parameter Weibull distributions under parametric uncertainty (when both parameters are unknown). The technique used here emphasizes pivotal quantities relevant for obtaining tolerance factors and is applicable whenever the statistical problem is invariant under a group of transformations that acts transitively on the parameter space. It does not require the construction of any tables and is applicable whether the experimental data are complete or Type II censored. The exact tolerance limits on order statistics associated with sampling from underlying distributions can be found easily and quickly making tables, simulation, Monte Carlo estimated percentiles, special computer programs, and approximation unnecessary. The proposed technique is based on a probability transformation and pivotal quantity averaging. It does not in need to make any assumption concerning the statistical functional form for the tolerance limit, is conceptually simple and easy to use. The scientific literature does not contain an analytical methodology for constructing exact γ-content tolerance limits with expected $(1-\alpha)$-confidence on future order statistics coming from an extreme-value or Weibull distribution. One reason is that the theoretical concept and computational complexity of the tolerance limits is significantly more difficult than that of the standard confidence and prediction limits. However, in the literature there are several known methods for constructing $(1\text{-}\alpha)$-prediction limits (in terms of this article, tolerance limits with expected $(1\text{-}\alpha)$-confidence) on future order statistics coming from the two-parameter Weibull distribution. Therefore, finally, we give numerical examples, where the $(1\text{-}\alpha)$-prediction limits obtained by using the known methods are compared with the results obtained through the proposed analytical methodology, which is illustrated in terms of the extreme-value and two-parameter Weibull distributions. Analytical formulas for the tolerance limits are available in the scientific literature for only simple cases, for example, for the upper or lower tolerance

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limit for a univariate normal population. Thus it becomes necessary to use new methods in order to derive exact statistical tolerance limits for many populations. The proposed, in this article, technique of intelligent constructing exact statistical γ-content tolerance limits with expected $(1-\alpha)$-confidence, which are obtained here in terms of the two-parameter Weibull and extreme-value distributions, represents a novelty in the theory of statistical decisions.

BACKGROUND

In this paper, two types of statistical tolerance limits are defined: i) γ-content tolerance limits with expected $(1-\alpha)$-confidence on future outcomes, ii) tolerance limits with expected $(1-\alpha)$-confidence on future outcomes. To be specific, let γ denote a proportion between 0 and 1. Then one-sided statistical γ-content tolerance limit with expected $(1-\alpha)$-confidence is determined to capture a proportion γ or more of the population, with a given expected confidence level $1-\alpha$. For example, an upper statistical γ-content tolerance limit with expected $(1-\alpha)$-confidence on future outcomes from a univariate population is such that with the given expected confidence level $1-\alpha$, a specified proportion γ or more of the population will fall below the limit. A lower statistical γ-content tolerance limit with expected $(1-\alpha)$-confidence satisfies similar conditions. An upper statistical tolerance limit with expected $(1-\alpha)$-confidence is determined so that the expected proportion of the population failing below the limit is $(1-\alpha)$. A lower statistical tolerance limit with expected $(1-\alpha)$-confidence satisfies similar conditions. The statistical γ-content tolerance limit with expected $(1-\alpha)$-confidence seems to be more useful than the statistical tolerance limit with expected $(1-\alpha)$-confidence but is relatively difficult to construct.

The logical purpose for a tolerance limit must be the prediction of future outcomes for some (say, stochastic) process. Tolerance (prediction) limits enjoy a fairly rich history in the scientific literature and have a very important role in engineering and manufacturing applications. Patel (1986) provides a review (which was fairly comprehensive at the time of publication) of tolerance intervals (limits) for many distributions as well as a discussion of their relation with confidence intervals (limits) for percentiles. Dunsmore (1978) and Guenther, Patil, and Uppuluri (1976) both discuss 2-parameter exponential tolerance intervals (limits) and the estimation procedure in greater detail. Engelhardt and Bain (1978) discuss how to modify the formulas when dealing with type II censored data. Guenther (1972) and Hahn and Meeker (1991) discuss how one-sided tolerance limits can be used to obtain approximate two-sided tolerance intervals by applying Bonferroni's inequality. In Nechval et al. (1999, 2004, 2014a, 2016a, 2016b, 2016c, 2017, 2018a, 2019), the exact statistical tolerance and prediction limits are discussed under parametric uncertainty of underlying models.

In contrast to other statistical limits commonly used for statistical inference, the γ-content tolerance limits with expected $(1-\alpha)$-confidence (especially for the order statistics) are used relatively rarely. One reason is that the theoretical concept and computational complexity of the γ-content tolerance limits with expected $(1-\alpha)$-confidence is significantly more difficult than that of the standard confidence and prediction limits. Thus it becomes necessary to use the innovative approaches which will allow one to construct tolerance limits on future order statistics for many populations.