Chapter 6
Quantum Wavelet Packet Transforms

ABSTRACT

Quantum wavelet packet transform (QWPT) may play an important role in quantum information processing. In this chapter, the authors design quantum circuits of a generalized tensor product (GTP) and a perfect shuffle permutation (PSP). Next, they propose multi-level and multi-dimensional (1D, 2D and 3D) QWPTs, including Haar QWPT (HQWPT), D4 QWPT (DQWPT) based on the periodization extension and their inverse transforms for the first time, and prove the correctness based on the GTP and PSP. Furthermore, they analyze the quantum costs and the time complexities of the proposed QWPTs and obtain precise results. The time complexities of HQWPTs is at most six basic operations on 2n elements, which illustrates high efficiency of the proposed QWPTs.

INTRODUCTION

Quantum wavelet packet transform (QWPT) can be classified with one-dimensional or multi-dimensional according to the types of data it acts on are 1-dimension or multi-dimension. The QWPT can be used repeatedly. The level of QWPT describes the number of times QWPT acts on data. Two 1-dimensional QWPTs have been developed, which are the single-level 1-dimensional Haar QWPT, and the single-level 1-dimensional Daubechies D4 QWPT, respectively (Hoyer, 1997; Fijany & Williams, 1998; Terraneo

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& Shepelyansky, 2003; Klappenecker, 1999). In addition, multi-level and multi-dimensional QWPTs were proposed (Li, Fan, Xia, Song, & He, 2018).

This chapter introduces multi-level and multi-dimensional QWPTs (Li, Fan, Xia, Song, & He, 2018), and uses 2-dimensional quantum wavelet packet transforms to implement quantum image compression (Li, Zhu, Zhou, Li, Song, & Ian, 2014). Meanwhile, simulations of multi-level and multi-dimensional QWPTs are given.

1-DIMENSIONAL QUANTUM WAVELET PACKET TRANSFORMS

In this section, multi-level 1-dimensional quantum wavelet packet transforms (1D-QWPTs) are introduced. These 1D-QWPTs include 1-dimensional general quantum wavelet packet transform, 1-dimensional Haar quantum wavelet packet transform (1D-HQWPT), and 1-dimensional Daubechies D4 quantum wavelet packet transform (1D-D4QWPT).

1-Dimensional General Quantum Wavelet Packet Transform

Let $W_{2^n}^0 = W_{2^n}$ be a wavelet kernel matrix. Then, the (k+1)-level iteration of a discrete wavelet packet transform is defined by (Ruch, & Van Fleet, 2011)

$$\begin{cases} Z_{2^n}^k = W_{2^n}^k W_{2^n}^{k-1} \ldots W_{2^n}^1 W_{2^n}^0, \\ W_{2^n}^j = Diag(W_{2^{n-j}}, W_{2^{n-j}}, \ldots, W_{2^{n-j}}), \end{cases} \tag{7.1}$$

where $Diag(W_{2^{n-j}}, W_{2^{n-j}}, \ldots, W_{2^{n-j}})$ with j=1,…,k is a matrix with 2^j blocks of $W_{2^{n-j}}$ on the main diagonal and zeros elsewhere. We infer the following equations,

$$\begin{cases} W_{2^n}^j = Diag(W_{2^{n-1}}^{j-1}, W_{2^{n-1}}^{j-1}), \\ Z_{2^n}^k = Diag(Z_{2^{n-1}}^{k-1}, Z_{2^{n-1}}^{k-1}) W_{2^n}. \end{cases} \tag{7.2}$$

According to the generalized tensor product in (5.12), we have