Chapter 6 Quantum Wavelet Packet Transforms

ABSTRACT

Quantum wavelet packet transform (QWPT) may play an important role in quantum information processing. In this chapter, the authors design quantum circuits of a generalized tensor product (GTP) and a perfect shuffle permutation (PSP). Next, they propose multi-level and multi-dimensional (1D, 2D and 3D) QWPTs, including Haar QWPT (HQWPT), D4 QWPT (DQWPT) based on the periodization extension and their inverse transforms for the first time, and prove the correctness based on the GTP and PSP. Furthermore, they analyze the quantum costs and the time complexities of the proposed QWPTs and obtain precise results. The time complexities of HQWPTs is at most six basic operations on 2n elements, which illustrates high efficiency of the proposed QWPTs.

INTRODUCTION

Quantum wavelet packet transform (QWPT) can be classified with onedimensional or multi-dimensional according to the types of data it acts on are 1-dimension or multi-dimension. The QWPT can be used repeatedly. The level of QWPT describes the number of times QWPT acts on data. Two 1-dimensional QWPTs have been developed, which are the single-level 1-dimensional Haar QWPT, and the single-level 1-dimensional Daubechies D4 QWPT, respectively (Hoyer, 1997; Fijany & Williams, 1998; Terraneo

DOI: 10.4018/978-1-7998-3799-2.ch006

& Shepelyansky, 2003; Klappenecker, 1999). In addition, multi-level and multi-dimensional QWPTs were proposed (Li, Fan, Xia, Song, & He, 2018).

This chapter introduces multi-level and multi-dimensional QWPTs (Li, Fan, Xia, Song, & He, 2018), and uses 2-dimensional quantum wavelet packet transforms to implement quantum image compression (Li, Zhu, Zhou, Li, Song, & Ian, 2014). Meanwhile, simulations of multi-level and multi-dimensional QWPTs are given.

1-DIMENSIONAL QUANTUM WAVELET PACKET TRANSFORMS

In this section, multi-level 1-dimensional quantum wavelet packet transforms (1D-QWPTs) are introduced. These 1D-QWPTs include 1-dimensional general quantum wavelet packet transform, 1-dimensional Haar quantum wavelet packet transform (1D-HQWPT), and 1-dimensional Daubechies D4 quantum wavelet packet transform (1D-D4QWPT).

1-Dimensional General Quantum Wavelet Packet Transform

Let $W_{2^n}^0 = W_{2^n}$ be a wavelet kernel matrix. Then, the (*k*+1)-level iteration of a discrete wavelet packet transform is defined by (Ruch, & Van Fleet, 2011)

$$\begin{cases} Z_{2^{n}}^{k} = W_{2^{n}}^{k} W_{2^{n}}^{k-1} \dots W_{2^{n}}^{1} W_{2^{n}}^{0}, \\ W_{2^{n}}^{j} = Diag(W_{2^{n-j}}, W_{2^{n-j}}, \dots, W_{2^{n-j}}), \end{cases}$$
(7.1)

where $Diag(W_{2^{n-j}}, W_{2^{n-j}}, ..., W_{2^{n-j}})$ with j=1,...,k is a matrix with 2^{j} blocks of $W_{2^{n-j}}$ on the main diagonal and zeros elsewhere. We infer the following equations,

$$\begin{cases} W_{2^{n}}^{j} = Diag(W_{2^{n-1}}^{j-1}, W_{2^{n-1}}^{j-1}), \\ Z_{2^{n}}^{k} = Diag(Z_{2^{n-1}}^{k-1}, Z_{2^{n-1}}^{k-1})W_{2^{n}}. \end{cases}$$
(7.2)

According to the generalized tensor product in (5.12), we have

23 more pages are available in the full version of this document, which may be purchased using the "Add to Cart" button on the publisher's webpage: <u>www.igi-</u> <u>global.com/chapter/quantum-wavelet-packet-</u> <u>transforms/261478</u>

Related Content

Online Healthcare Communities of Practice: Identifying the Critical Success Factors

Haitham Alali (2016). International Journal of Computers in Clinical Practice (pp. 1-14).

www.irma-international.org/article/online-healthcare-communities-of-practice/152589

Introduction to Data Mining

(2022). Implementation of Machine Learning Algorithms Using Control-Flow and Dataflow Paradigms (pp. 1-13). www.irma-international.org/chapter/introduction-to-data-mining/299338

An Ultra-Fast Method for Clustering of Big Genomic Data

Billel Kenidraand Mohamed Benmohammed (2020). *International Journal of Applied Metaheuristic Computing (pp. 45-60).*

www.irma-international.org/article/an-ultra-fast-method-for-clustering-of-big-genomicdata/239557

Neuropsychological and Cognitive Control Deficits in Depression

Meenakshi Banerjee (2022). *Bio-Inspired Algorithms and Devices for Treatment of Cognitive Diseases Using Future Technologies (pp. 94-116).* www.irma-international.org/chapter/neuropsychological-and-cognitive-control-deficits-indepression/298807

A New Mechanical Algorithm for Calculating the Amplitude Equation of the Reaction-Diffusion Systems

Houye Liuand Weiming Wang (2012). International Journal of Computational Models and Algorithms in Medicine (pp. 21-28).

www.irma-international.org/article/a-new-mechanical-algorithm-for-calculating-the-amplitudeequation-of-the-reaction-diffusion-systems/101425