Chapter 22
Discrete Time Chaotic Maps With Application to Random Bit Generation

Lazaros Moysis
https://orcid.org/0000-0002-5652-2532
Laboratory of Nonlinear Systems – Circuits & Complexity, Physics Department, Aristotle University of Thessaloniki, Greece

Ahmad Taher Azar
https://orcid.org/0000-0002-7869-6373
Robotics and Internet-of-Things Lab (RIOTU), Prince Sultan University, Riyadh, Saudi Arabia

Aleksandra Tutueva
https://orcid.org/0000-0002-9747-1962
Youth Research Institute, Saint Petersburg Electrotechnical University "LETI", Russia

Denis N. Butusov
Youth Research Institute, Saint Petersburg Electrotechnical University "LETI", Russia

Christos Volos
Laboratory of Nonlinear Systems – Circuits & Complexity, Physics Department, Aristotle University of Thessaloniki, Greece

ABSTRACT

Chaotic behavior is a term that is attributed to dynamical systems whose solutions are highly sensitive to initial conditions. This means that small perturbations in the initial conditions can lead to completely different trajectories in the solution space. These types of chaotic dynamical systems arise in various natural or artificial systems in biology, circuits, engineering, computer science, and more. This chapter reports on some new chaotic discrete time two-dimensional maps that are derived from simple modifications to the well-known Hénon, Lozi, Sine-Sine, and Tinkerbell maps. Numerical simulations are carried out for different parameter values and initial conditions, and it is shown that the mappings either diverge to infinity or converge to attractors of many different shapes. The application to random bit generation is then considered using a collection of the proposed maps by applying a simple rule. The resulting bit generator successfully passes all statistical tests performed.

DOI: 10.4018/978-1-7998-5788-4.ch022

1. INTRODUCTION

By chaos and chaotic systems, we generally refer to dynamical systems whose behavior is highly sensitive to initial conditions. That is, small changes in the initial conditions of such a system will lead to completely different trajectories in the solution space, a phenomenon that is widely referred to as the butterfly effect, see for example Alligood et al. (1996). Thus, despite such systems being deterministic in nature with no stochastic parameters, their complex nature constitutes their behavior hard to predict and control. This happens due to the presence of nonlinear terms in the differential equations that describe the system.

Chaotic systems were introduced in the seminal work of Lorenz (1963), that introduced a three-dimensional weather model that described forced dissipative hydrodynamic flow. Since then, chaos theory as a discipline has vastly expanded and chaotic systems have been extensively studied over the last 40 years. The advancements in this area have been greatly influenced by the advancement of technology and personal computers, which makes it easier to follow the trajectories of chaotic systems for a longer time period and with a higher precision. Thus, the theoretical analysis of chaos has been assisted by the numerical simulations of such chaotic systems, made possible by the use of computers.

Out of the many chaotic systems, the ones that seem to be the most interesting are those that exhibit an attracting behavior. By that, we refer to systems whose trajectories converge with time to a set of values, called an *attractor* (Wang et al., 2017; Volos et al., 2018). This can be a single point, a curve, a surface or a manifold. For example, in a system with a spherical bowl and a rolling marble, the bottom of the bowl is called a *fixed-point* attractor. The attractor is called *strange* if it is a set with fractal structure, see Grassberger and Procaccia (2004); Ott (1981). Examples include the aforementioned Lorenz system, as well as the *Hénon, Lozi* and *Tinkerbell* maps that will be introduced in the next sections. It should also be noted that a system may have different attractors depending on the choice of initial conditions.

Chaotic systems have found their place in applications spanning a wide variety of natural and artificial systems. These include oscillators (Heagy, 1992; Kengne et al. 2012; Sharma et al. 2012), secure communications (Murali & Lakshmanan, 1998; Zaher, & Abu-Rezq, 2011), controller design (dos Santos Coelho & Mariani, 2012), lasers (Li et al. 2014; Yuan et al. 2014), chemical reactions (Gaspard, 1999), pseudo-random number generation (Stoyanov & Kordov, 2015), optical ring phase resonators (Aboites et al. 2009), biology (Das et al. 2014; Kyriazis, 1991) medicine (Qu, 2011, Witte & Witte, 1991), finance (Guegan, 2009; Nosoohi & Parvizian, 2013) and more.

A major task in the control of chaotic systems, is the one of finding a feedback control of the general form $u(t)=k(x(t))$, where u the input, x the state of the system and k the feedback law, in order to stabilize the system around its unstable equilibrium. Techniques include active control, adaptive control, sliding mode control, fuzzy logic control (Alain et al., 2017; Vaidyanathan & Azar, 2015a, 2016a,b,c).

Chaos synchronization is another significant problem, which rises when two or more chaotic systems are coupled, or when a chaotic system, called the *master system*, drives another chaotic system, called the *slave system* (El-Ghazaly; 2012; Azar et al., 2017; Ouannas et al., 2017a,b,c,d,e; Singh et al., 2017, 2018a,b,c; Khan et al., 2020a,b; Longge & Xiangjie, 2013;). Due to the aforementioned butterfly effect in chaotic systems which causes the exponential divergence of the trajectories of two identical systems with nearly the same initial conditions, their synchronization is a difficult task and is in itself a subject of extensive research. As with the feedback control, various techniques have been established for chaos synchronization. The choice of the technique used depends on the type of the system and the available resources. For example, the *active control method* (Rafikov & Balthazar, 2008; Longe et al., 2013) is

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