



# Optimal Solutions for a Multi-Product, Multi-Echelon Production and Replenishment System

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## ABSTRACT

*The effective coordination is a key element in the success of many cooperative supply chains. All production, distribution and supply must be adequately synchronized in order to satisfy the customer needs and at the same time optimizing the operational costs. This article presents a multi-product, multi-echelon inventory system which comprises one manufacturer, a number of distribution centers and a number of retailers which are dependent of such distribution centers. The coordination and collaboration is achieved through a carefully designed replenishment policy. The near-optimal order quantities for each of the supply chain agents are calculated with a mathematical model in which the integrality constraints are relaxed. A number of instances were generated and tested. The results show the validity of the proposed approach. [Article copies are available for purchase from InfoSci-on-Demand.com]*

*Keywords: Coordinated Replenishment; Multi-Echelon Inventory Systems; Supply Chain*

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## INTRODUCTION

Starting in the 1960s, a number of academic works have focused in serial and distributed multi-echelon structures and in both deterministic and stochastic environments. Clark and Scarf (1960, 1962) developed a seminal work in which they presented optimal policies for a serial multi-echelon system. Schwartz (1973) studied a deterministic model having one warehouse and  $N$  retailers and showed that optimal policies can be very difficult to find.

The concept of linked stationary policies was presented by Schwartz (1973) and Love (1972). Many heuristics have been developed to extend the classical EOQ (Economic Order Quantity) model to multi-echelon systems. A disadvantage is that most of them lack of verifiable lower bounds (Muckstad and Roundy, 1987) as in the case of Blackburn and Millen (1982), Williams (1981), Crowston and Wagner (1973), Crowston. Wagner and Henshaw (1972), Crowston, Wagner and Williams (1973),

Graves (1981), Jensen and Khan (1972) and McLaren (1976).

In the context of minimizing costs in the case of one warehouse and  $N$  retailers, Graves and Schwartz (1977) developed a branch and bound-like technique that finds optimal solutions for linked multi-echelon systems. A disadvantage is that the number of branches grows exponentially with the number of retailers. Muckstad and Roundy (1987) proposed an efficient method for minimizing costs in a multi-product, one warehouse and  $N$  retailers problem. The algorithm runs in  $O(N \log N)$  time and its results fall within 2% of the optimal solution. More recently Abdul-Jalbar et al (2006) presented another heuristic for a single product, one warehouse and  $N$  retailers problem. Their results are within 1% of the optimal solution.

Integrated inventory models in the case of one vendor and multiple buyers have been presented by Lu (1995), Banerjee (1986), Goyal (2000), Lal and Staelin (1984), Lee and Rossemblatt (1986), Kim and Hwang (1989), where a number of quantity discount schemes and lot for lot policies are considered. Goyal (1977) suggested a Joint Economic Lot Sizing (JELS) model whose objective was to minimize total costs in the vendor-buyer chain. Banerjee (1986) generalized such a model by incorporating finite production rates.

Later Goyal (1998) extended the Banerjee (1986) model by removing the lot for lot production for the vendor. Wee and Yang (2004) revised the Goyal (1998) model and found both optimal and heuristic solutions in a network of manufacturer – distributors and retailers using the coordinated replenishment model in a supply chain. This model showed significant cost reductions compared to the Goyal (1998) model. For the stochastic demand case, studies began with the works of Clark and Scarf (1960) and later with Federgruen and Zipkin (1984), Rosling (1989) and Chen and Zheng (1994). Literature reviews on this topic can be found in Federgruen (1993), and Zipkin (2000). The basic stochastic model was named as METRIC and was developed by Sherbrooke (1968). Many

stochastic models for the case of one warehouse and  $N$  retailers are based on this approach (Axsäter, 1997; 2005), Axsäter and Juntti (1996) and Cheng and Zheng (1994; 1998).

The joint coordination of inventories in the deterministic multi-product case has been studied in different approaches. The basic model is the Economic Lot Sizing and Scheduling Problem (ELSP) in which the manufacturer produces a number of items having a resource constraint (i.e. only one item can be produced at a time). Both the production and demand rates are known. The first papers date from the 1960s (Hanssman, 1962; Maxwell; 1964). More recent developments include Dobson (1987; 1992), Jones and Inmann (1989), Roundy (1989), Gallego (1990) and Zipkin (1991). Davis (1990) proposed an enumerative model which combines non-linear optimization, a heuristic procedure and a Mixed Integer Programming (MIP) model. The ELSLP model has also been extended to handle backorders (Gallego and Roundy, 1992; Dobson and Yano, 1990). Several extensions to serial systems have been studied (Dobson and Yano, 1990; El-Najdawi, 1992; El-Najdawi and Kleindorfer, 1993).

In this article we consider a collaborative system in which inventories are coordinated among one manufacturer, multiple distributors and multiple retailers. Our work allows efficient calculations of near-optimal solutions through a simple heuristic local search procedure. The objective function is the total inventory cost. The proposed approach extends the single product model presented in Wee and Yang (2004). Our model considers multiple products, finite production rates and setup times. The synchronization in the supply chain is achieved by integrating the ELSLP model into the coordinated replenishment approach.

## PROBLEM FORMULATION AND ASSUMPTIONS

Our proposed model is based on previous work from the literature with constraints not

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