

Chapter XXVIII

Fractal Geometry and Computer Science

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ABSTRACT

Fractal geometry can help us to describe the shapes in nature (e.g., ferns, trees, seashells, rivers, mountains) exceeding the limits imposed by Euclidean geometry. Fractal geometry is quite young: The first studies are the works by the French mathematicians Pierre Fatou (1878-1929) and Gaston Julia (1893-1978) at the beginning of the 20th century. However, only with the mathematical power of computers has it become possible to realize connections between fractal geometry and other disciplines. It is applied in various fields now, from biology to economy. Important applications also appear in computer science because fractal geometry permits us to compress images, and to reproduce, in virtual reality environments, the complex patterns and irregular forms present in nature using simple iterative algorithms executed by computers. Recent studies apply this geometry to controlling traffic in computer networks (LANs, MANs, WANs, and the Internet). The aim of this chapter is to present fractal geometry, its proper-

ties (e.g., self-similarity), and their applications in computer science.

INTRODUCTION

Fractal geometry is a recent discovery. It is also known as Mandelbrot's geometry in honor of its father, the Polish-born Franco-American mathematician Benoit Mandelbrot, who showed how fractals can occur in many different places in both mathematics and elsewhere in nature.

Fractal geometry is now recognized as the true geometry of nature. Before Mandelbrot, mathematicians believed that most of the patterns of nature were far too irregular, complex, and fragmented to be described mathematically. Mandelbrot's geometry replaces Euclidian geometry, which had dominated our mathematical thinking for thousands of years.

The *Britannica Concise Encyclopedia* ("Fractal Geometry," 2007) introduces fractal geometry as follows:

In mathematics, the study of complex shapes with the property of self-similarity, known as fractals. Rather like holograms that store the entire image in each part of the image, any part of a fractal can be repeatedly magnified, with each magnification resembling all or part of the original fractal. This phenomenon can be seen in objects like snowflakes and tree bark.... This new system of geometry has had a significant impact on such diverse fields as physical chemistry, physiology, and fluid mechanics; fractals can describe irregularly shaped objects or spatially nonuniform phenomena that cannot be described by Euclidean geometry."

The multiplicity of the application fields had a central role in the diffusion of fractal geometry (Barnsley, Saupe, & Vrscay, 2002; Eglash, 1999; Mandelbrot, 1982; Nonnenmacher, Losa, Merlini, & Weibel, 1994; Sala, 2004, 2006; Vyzantiadou, Avdelas, & Zafiropoulos, 2007).

BACKGROUND: WHAT IS A FRACTAL?

A fractal could be defined as a rough or fragmented geometric shape that can be subdivided in parts, each of which is approximately a reduced-size copy of the whole (Mandelbrot, 1988). *Fractal* is a term coined by Benoit Mandelbrot (born 1924) to denote the geometry of nature, which traces inherent order in chaotic shapes and processes. The term derived from the Latin verb *frangere*, *to break*, and from the related adjective *fractus*, *fragmented and irregular*. This term was created to differentiate pure geometric figures from other types of figures that defy such simple classification. The acceptance of the word fractal was dated in 1975. When Mandelbrot presented the list of publications between 1951 and 1975, the date when the French version of his book was published, people were surprised by the variety of the studied fields: linguistics, cosmology, economy, games theory, turbulence, and noise on telephone lines

(Mandelbrot, 1975). Fractals are generally self-similar on multiple scales. So, all fractals have a built-in form of iteration or recursion. Sometimes the recursion is visible in how the fractal is constructed. For example, Koch's snowflake, Cantor's set, and Sierpinski's triangle are generated using simple recursive rules. Self-similarity, iterated function systems, and the Lindenmayer System are applied in different fields of computer science (e.g., in computer graphics, virtual reality, and traffic control for computer networks).

Self-Similarity

Self-similarity, or invariance against changes in scale or size, is a property by which an object contains smaller copies of itself at arbitrary scales. Mandelbrot (1982, p. 34) defined self-similarity as follows: "When each piece of a shape is geometrically similar to the whole, both the shape and the cascade that generate it are called self-similar."

A fractal object is self-similar if it has undergone a transformation whereby the dimensions of the structure were all modified by the same scaling factor. The new shape may be smaller, larger, translated, and/or rotated. Similar means that the relative proportions of the shapes' sides and internal angles remain the same. As described by Mandelbrot (1982), this property is ubiquitous in the natural world. Oppenheimer (1986) used the term fractal, exchanging it with self-similarity, and he affirmed that the geometric notion of self-similarity is evolving in a paradigm for modeling the natural world, in particular in the world of botany.

Self-similarity appears in objects as diverse as leaves, mountain ranges, clouds, and galaxies. Figure 1a shows a snowflake that is an example of self-similarity in nature. Figure 1b illustrates Koch's snowflake; it is built starting from an equilateral triangle, removing the inner third of each side, building another equilateral triangle at the location where the side was removed, and

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