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Towards the Fused Bayesian-Regularization Method for Computer-Aided Enhancement of the Remotely Sensed Imagery

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ABSTRACT

In this paper, we address a new approach to solving the ill-posed nonlinear inverse problem of reconstruction of the radar images of the wavefield sources distributed in the environment via processing the remotely sensed data signals distorted in the stochastic measurement channel. By exploiting the idea of combining the Bayesian estimation theory and descriptive regularization techniques we address a new fused Bayesian-regularization method for image enhancement as it is required for computer-aided imagery with remotely sensed data.

I. INTRODUCTION

In this paper, we address a new approach to computer-aide enhancement of radar images (RI) stated and treated as an ill-posed inverse problem of reconstruction of the spatial spectrum pattern (SSP) of the wavefield sources scattered from the probing surface via processing the remotely sensed data signals distorted in the stochastic measurement channel. We propose a new fused Bayesian-regularization (FBR) method that combines the Bayesian inference paradigm [1], [2] with the descriptive regularization techniques [3], [5] for scattering inverse problems solution. With this method, we present the FBR-based interpretation of a family of the conventional array imaging algorithms and propose some their modifications to enhance the resolution performances of the computer-reconstructed radar images.

II. SSP ESTIMATION AS AN INVERSE PROBLEM

A. Problem Statement

Consider a coherent RI experiment in a random medium and the narrowband assumption [1], [4] that enables one to model the backscattered field of the remotely sensed object associated with the probing surface X ‘ $\mathbf{x}$ by imposing its time invariant complex scattering function $e(\mathbf{x})$ over the object scene X. The measurement data wavefield $u(\mathbf{y}) = s(\mathbf{y}) + n(\mathbf{y})$ consists of the echo signals s and additive noise n, and is available for observations and recordings within the prescribed time-space observation domain $Y = T´P$, where $\mathbf{y} = (t, \mathbf{p})^T$ defines the time-space points in Y. The model of the observation wavefield u is defined by specifying the stochastic equation of observation of an operator form [1]: $u = Se + n$; $e \in$ E; u, $n \in$ U; S: E $\rightarrow$U, in the Gilbert signal spaces E and U with the metric structures induced by the inner products, $[u_1, u_2]_U = \int_Y u_1(\mathbf{y})u_2^*(\mathbf{y})d\mathbf{y}$, and $[e_1, e_2]_E = \int_X e_1(\mathbf{x})e_2^*(\mathbf{x})d\mathbf{x}$, respectively. The operator model of the stochastic equation of observation (EO) in the conventional integral form [1], [3] may be rewritten as

$$u(\mathbf{y}) = (Se(\mathbf{x}))(\mathbf{y}) = \int_X S(\mathbf{y},\mathbf{x})\, e(\mathbf{x})d\mathbf{x} + n(\mathbf{y}) = \int_X S_0(\mathbf{y},\mathbf{x})\, e(\mathbf{x})d\mathbf{x} + \int_X S_\mu(\mathbf{y},\mathbf{x})\, e(\mathbf{x})d\mathbf{x} + n(\mathbf{y})\,. \qquad (1)$$

The random functional kernel $S(\mathbf{y}, \mathbf{x})$ of the operator S given by (1) defines the signal wavefield formation model. Its mean $S_0(\mathbf{y}, \mathbf{x})$ is referred to as the regular signal formation operator (SFO) in the data measurement channel defined by the time-space modulation of signals employed in a particular radar system [3], [4]. The random variation about the mean $S_m(\mathbf{y}, \mathbf{x}) = m(\mathbf{y}, \mathbf{x})S_0(\mathbf{y}, \mathbf{x})$ models the stochastic perturbations of the wavefield at different propagation paths, where $m(\mathbf{y}, \mathbf{x})$ is the zero-mean multiplicative noise specified by the propagation properties of the medium [1]. All the fields e, n, u in (1) are assumed to be zero-mean complex valued Gaussian random fields. Next, we assume an incoherent nature of the backscattered field $e(\mathbf{x})$. This is naturally inherent to the RI experiments [1], [3] and leads to the δ-form of the object field correlation function, $R_e(\mathbf{x}_1,\mathbf{x}_2) = B(\mathbf{x}_1)d(\mathbf{x}_1- \mathbf{x}_2)$, where $e(\mathbf{x})$ and $B(\mathbf{x}) = <|e(\mathbf{x})|^2>$ are referred to as a random complex scattering function of the extended object (probing surface) and its average power scattering function or spatial spectrum pattern (SSP), respectively. The inverse problem of SSP reconstruction is to derive an estimate $\hat{B}(\mathbf{x})$ (referred to as the desired radar image) by processing the available finite dimensional array (synthesized array) measurements of the data wavefield $u(\mathbf{y})$.

B. Projection Model of the Data Measurements

Viewing it as an approximation problem leads one to the projection concept for reduction of the data field $u(\mathbf{y})$ to the M-D vector $\mathbf{U}$ of sampled spatial-temporal data recordings

$$\mathbf{U} = \mathbf{SE} + \mathbf{N}\,, \qquad (2)$$

where $\mathbf{E}$, and $\mathbf{U}$ are the zero-mean vectors composed of the coefficients $E_k = [e, g_k]_E$; $k = 1, \ldots, K$ and $U_m = [u, h_m]_U$; $m = 1, \ldots, M$, of the finite dimensional approximations of the corresponding fields e, u in the approximation (measurement) spaces $U_{(M)}$ = span$\{h_m\}$ and $E_{(K)}$ = span$\{g_k\}$ spanned by the properly selected basis functions $\{h_m\}$ and $\{g_k\}$ [5]. In (2), matrix $\mathbf{S}$ corresponds to the $M´K$ approximation of the regular SFO S_0 [5], and $\mathbf{N}$ is the composite noise vector that accumulates the K-D approximation of the signal-dependent noise component given by the second integral in (1). Physically, the complex conjugate set {$h^*_m(\mathbf{y})$} is specified by a composition of the antenna element tapering functions {$\tau_l(\mathbf{p})$; l = 1, …, L} (that we admit to be either identical or different for the different elements of the L-D array), and the impulse response functions {$\chi_i(t)$; i = 1, …, I} of the I sampling filters in the corresponding spatial receive channels (as well identical or different) ordered by multi-index $m = (l, i) = 1, …, M = L´I$. In practice, the antenna elements are distanced in space (do not overlap), i.e. the tapering functions {$t_l(\mathbf{p})$} have the distanced supports in $P \ni \mathbf{p}$, thus, they compose a set of orthogonal functions. The same assumption of orthogonality is usually valid for the sampling filters {$\chi_i(t)$}, $t \in T$, in which case the dual basis [5] {$\phi_m(\mathbf{y})$} is simply the properly normalized set of {$h_m(\mathbf{y})$}, i.e. {$\phi_m(\mathbf{y}) = ||h_m(\mathbf{y})||^{-2}h_m(\mathbf{y})$; m = 1, …, M}.

The same projection formalism is valid for the set of basis functions {g_k} with the corresponding dual basis {$j_k(\mathbf{x})$} [5].

Vectors **E**, **N** and **U** are characterized by the correlation matrices $\mathbf{R_E} = \mathbf{D} = \mathbf{D}(\mathbf{B}) = \text{diag}(\mathbf{B})$ (a diagonal matrix with vector **B** at its main diagonal), $\mathbf{R_N}$, and $\mathbf{R_U} = \mathbf{S}\mathbf{R_E}\mathbf{S}^+ + \mathbf{R_N}$, respectively. (Recall that superscript $^+$ defines the Hermitian conjugate when stands with a matrix or vector). The vector, **B**, is composed of the elements $B_k = \langle E_k E_k^* \rangle$; k = 1, …, K, and is referred to as a K-D vector-form approximation of the SSP.

We refer to the estimate $\hat{\mathbf{B}}$ as a discrete-form representation of the brightness image of the wavefield sources distributed in the environment remotely sensed with the array (or synthesized array) radar, in which case the continuous-form finite dimensional approximation of the estimate of the SSP distribution $\hat{B}_{(K)}(\mathbf{x})$ over the remotely sensed scene in a given spatial domain X ‘ $\mathbf{x}$ can be expressed as follows [5]

$$\hat{B}_{(K)}(\mathbf{x}) = \Sigma\, B_k |\varphi_k(\mathbf{x})|^2 = \boldsymbol{\varphi}^T(\mathbf{x})\text{diag}(\hat{\mathbf{B}})\boldsymbol{\varphi}(\mathbf{x}) \quad (3)$$

where $\mathbf{j}(\mathbf{x})$ represents a K-D vector composed of the dual basis functions {$j_k(\mathbf{x})$} = dual{g_k} in $E_{(K)}$.

Analyzing (3), one may deduce that in every particular measurement scenario (specified by the corresponding approximation spaces $U_{(M)}$ and $E_{(K)}$) one has to derive the estimate $\hat{\mathbf{B}}$ of a vector-form approximation of the SSP distribution over the scene that uniquely define the approximated continuous SSP estimate (3).

III. FBR STRATEGY FOR SSP ESTIMATION

In the descriptive statistical formalism, the desired SSP vector $\hat{\mathbf{B}}$ is recognized to be the vector of a main diagonal of the estimate of the correlation matrix $\mathbf{R_E}(\mathbf{B})$, i.e. $\hat{\mathbf{B}} = \{\hat{\mathbf{R}}_\mathbf{E}\}_{\text{diag}}$. Thus one can seek to estimate $\hat{\mathbf{B}} = \{\hat{\mathbf{R}}_\mathbf{E}\}_{\text{diag}}$ given the data correlation matrix $\mathbf{R_U}$ pre-estimated by some means, e.g. [2]

$$\hat{\mathbf{R}}_\mathbf{U} = \mathbf{Y} = \underset{j\in J}{\text{aver}}\ \{\mathbf{U}_{(j)}\,\mathbf{U}^+_{(j)}\} \quad (4)$$

by determining the solution operator [that we also refer to as the image formation operator (IFO)] **F** such that

$$\hat{\mathbf{B}} = \{\hat{\mathbf{R}}_\mathbf{E}\}_{\text{diag}} = \{\mathbf{F}\mathbf{Y}\mathbf{F}^+\}_{\text{diag}}\ . \quad (5)$$

To optimize the search of **F** we propose here the following FBR strategy

$$\mathbf{F} \rightarrow \min_{\mathbf{F}}\{\hat{A}(\mathbf{F})\}, \quad (6)$$

$$\Re(\mathbf{F}) = \text{trace}\{(\mathbf{FS}-\mathbf{I})\mathbf{A}(\mathbf{FS}-\mathbf{I})^+\} + \alpha\,\text{trace}\{\mathbf{F}\mathbf{R_N}\mathbf{F}^+\}$$

that implies the minimization of the weighted sum of the systematic error (the first term in $\hat{A}(\mathbf{F})$) and fluctuation error (the second term in $\Re(\mathbf{F})$) in the desired estimate $\hat{\mathbf{B}}$ where the selection (adjustment) of the regularization parameter a and the weight matrix **A** provides the additional regularization degrees of freedom incorporating any descriptive properties of a solution if those are known a priori [4], [5].

In the case of the solution-dependent **A**, i.e. when **A** = **D**, the problem given by (6) is recognized to coincide with the Bayes minimum risk (BMR) strategy that optimally balances the spatial resolution and the noise energy in the resulting estimate [5]. In the case of other chosen $\mathbf{A} \neq \mathbf{D} = \text{diag}\{\mathbf{B}\}$, we regularize the absence of a priori knowledge about the SSP **B**, hence introduce additional degrees of freedom into the desired solution. That is why we address (6) as the FBR strategy.

III. UNIFIED FBR ESTIMATOR

Routinely solving the minimization problem (6) we obtain

$$\mathbf{F} = \mathbf{K}_{\mathbf{A},a}\mathbf{S}^+\mathbf{R}_\mathbf{N}^{-1} \quad (7)$$

where

$$\mathbf{K}_{\mathbf{A},a} = (\mathbf{S}^+\mathbf{R}_\mathbf{N}^{-1}\mathbf{S} + a\mathbf{A}^{-1})^{-1}\ . \quad (8)$$

For this solution operator (IFO) the minimal possible value $\hat{A}_{\min}(\mathbf{F}) = \text{tr}\{\mathbf{K}_{\mathbf{A},a}\}$ of the objective function $\hat{A}(\mathbf{F})$ is attained.

In the general case of arbitrary fixed a and **A**, the unified FBR estimator for the SSP vector becomes

$$\hat{\mathbf{B}}_{FBR} = \{\mathbf{K}_{\mathbf{A},a}\mathbf{S}^+\mathbf{R}_\mathbf{N}^{-1}\,\mathbf{Y}\,\mathbf{R}_\mathbf{N}^{-1}\,\mathbf{S}\mathbf{K}_{\mathbf{A},a}\}_{\text{diag}} = \{\mathbf{K}_{\mathbf{A},a}\,\underset{j\in J}{\text{aver}}\,\{\mathbf{Q}_{(j)}\mathbf{Q}^+_{(j)}\}\mathbf{K}_{\mathbf{A},a}\}_{\text{diag}} \quad (9)$$

where $\mathbf{Q}_{(j)} = \{\mathbf{S}^+\mathbf{R}_\mathbf{N}^{-1}\,\mathbf{U}_{(j)}\}$ is recognized to be an output of the matched spatial filtering algorithm with noise whitening that assumes the given noise correlation matrix $\mathbf{R_N}$. Although in practical scenarios the noise correlation matrix $\mathbf{R_N}$ is usually unknown, it is a common practice in such cases to accept the robust white noise model, i.e. $\mathbf{R}_\mathbf{N}^{-1} = (1/N_0)\mathbf{I}$, with the noise intensity N_0 pre-estimated by some means [2].

IV. FAMILY OF THE FBR-OPTIMIZED ESTIMATORS

A. Robust Spatial Filtering Algorithm

Consider the white zero-mean noise in observations and no preference to any prior model information, i.e putting **A** = **I**. Let the regularization parameter be adjusted as the inverse of the signal-to-noise ratio (SNR), e.g. $a = N_0/B_0$, where B_0 is the prior average gray level of the SSP. In that case the IFO **F** is recognized to be the Tikhonov-type robust spatial filter:

$$\mathbf{F}_{RSF} = \mathbf{F}^{(1)} = (\mathbf{S}^+\mathbf{S} + (N_0/B_0)\mathbf{I})^{-1}\mathbf{S}^+. \quad (10)$$

B. Matched Spatial Filtering Algorithm

Consider the model from the previous example for an assumption, $a >> ||\mathbf{S}^+\mathbf{S}||$, i.e. the case of a priority of suppression of the noise over the systematic error in the optimization problem (6). In this case, we can roughly approximate (10) as the matched spatial filter:

$$\mathbf{F}_{MSF} = \mathbf{F}^{(2)} \approx const \times \mathbf{S}^+. \quad (11)$$

C. Adaptive Spatial Filtering Algorithm

Consider the case of zero-mean noise with an arbitrary correlation matrix $\mathbf{R_N}$, equal importance of two error measures in (6), i.e. $a = 1$, and the solution dependent weight matrix $\mathbf{A} = \hat{\mathbf{D}} = \text{diag}(\hat{\mathbf{B}})$. In this case, the IFO **F** becomes the adaptive spatial filter:

$$\mathbf{F}_{ASF} = \mathbf{F}^{(3)} = \mathbf{H} = (\mathbf{S}^{+}\mathbf{R}_{\mathbf{N}}^{-1}\mathbf{S} + \hat{\mathbf{D}}^{-1})^{-1}\mathbf{S}^{+}\mathbf{R}_{\mathbf{N}}^{-1} . \qquad (12)$$

D. MVDR Version of the ASF Algorithm

As it was shown in [5], the solution operator defined by (12) can be represented also in another equivalent form

$$\mathbf{F}_{MVDR} = \mathbf{F}^{(4)} = \hat{\mathbf{D}}\,\mathbf{S}^{+}\mathbf{Y}^{-1} = (\mathbf{S}^{+}\mathbf{R}_{\mathbf{N}}^{-1}\mathbf{S} + \hat{\mathbf{D}}^{-1})^{-1}\mathbf{S}^{+}\mathbf{R}_{\mathbf{N}}^{-1} , \qquad (13)$$

in which case, the solution (9) can be expressed as [5]

$$\hat{\mathbf{B}}_{MVDR} = \{\hat{\mathbf{R}}_{\mathbf{E}}\}_{\text{diag}} = \{(\mathbf{S}^{+}\mathbf{Y}^{-1}\mathbf{S})^{-1}\}_{\text{diag}} \qquad (14)$$

that coincides with the well known minimum variance distortionless response (MVDR) method [2]

$$\hat{B}_k = (\mathbf{s}_k^{+}\mathbf{Y}^{-1}\mathbf{s}_k)^{-1}; \quad k = 1, \ldots, K. \qquad (15)$$

Here $\mathbf{s}_k$ represents the steering vector for the *k*th look direction, which in our notations is essentially the *k*th column vector of the SFO matrix $\mathbf{S}$.

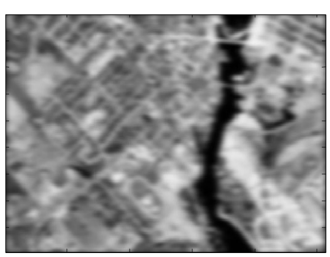
Examining the formulae (12) and (13) one may easily deduce that $\mathbf{F}^{(3)} = \mathbf{F}^{(4)}$. Thus, on one hand, the celebrated MVDR estimator (15) may be viewed as the convenient practical form of implementing the adaptive spatial filtering algorithm. On the other hand, it is obvious now that the MVDR beamformer may be considered as a particular case of the derived above unified FBR image formation algorithm (9) under the model assumptions: $\mathbf{A} = \text{diag}(\hat{\mathbf{B}})$, $a = 1$.

V. COMPUTER SIMULATIONS AND DISCUSSIONS

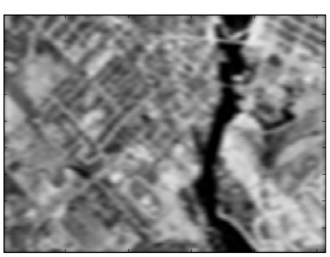
Figure 1. Simulation results: a) original scene; b) image formed applying the MSF method; c) image formed applying the RSF method; d) mage formed applying the MVDR version of the FBR method.

We simulated a conventional side-looking imaging radar (i.e. the array was constructed by the moving antenna) with the SFO factored along two axes in the image plane: the azimuth (horizontal axis) and the range (vertical axis). We considered the triangular radar range ambiguity function of 5 pixels width, and the $\exp(-ax^2)$ shape of the azimuth ambiguity function with parameter a adjusted to provide the 16 pixels width at 0.5 from its peak level. Figure 1.a shows the original scene of the 280-by-512 pixel format. The results of radar image formation for the 8% additive noise that employ the IFO given by (11), (10) and (12) are displayed in Figures 1.b, 1.c and 1.d, respectively. The advantage of the FBR-optimized imaging experiments (cases $\hat{\mathbf{B}}_{RSF}$ and $\hat{\mathbf{B}}_{ASF}$) over the conventional case $\hat{\mathbf{B}}_{MSF}$ is evident. Due to the performed regularized SFO inversions the resolution was improved in the both cases. For the statistically optimized estimator, $\hat{\mathbf{B}}_{ASF}$, in addition, the ringing effect was reduced, while the robust FBR-optimized estimator (RSF) with the IFO given by (10) requires substantially less computations.

(a)



(b)



(c)

(d)

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