Chapter 5
Mathematical Formalization and Abstract Description

ABSTRACT

The chapter is dedicated to the possibilities of formalization of an investigated object and its abstract description with appropriate mathematical means. At the beginning, the need to decompose complex systems is discussed to facilitate the following model study. The appropriate elements of a standard formal theory, applied in mathematical formalization and the abstract description of the investigated object, are formulated. A major place is devoted to the application of discrete and stochastic structures for the formalization of computer systems and processes for the purposes of further modeling. In the part of discrete means, the possibilities of formalization based on set theory, binary and weighted graphs, theory of finite automata are discussed. Some examples of specific applications are also given. The discussion of stochastic means is mainly focused on the application of elements of probability theory (random processes, streams of random events), the theory of Markov processes (in particular the application of Markov chains), and the theory of queues.

1. DECOMPOSITION OF COMPLEX SYSTEMS AND FORMALIZTION

Modeling is an abstraction that seeks to provide insight into processes and structures in the real environment. Technological development has allowed many types of modeling to be developed – from completely intuitive models to highly controlled ones. The basis of each approach is the use of specific terms and semantic tools that define the model language with rules for its application. Defining such an alphabet can be done based on natural language symbols, but assigning meaning to linguistic elements can lead to ambiguity and changing syntactic rules. The introduction of strict formal languages that mathematics offers, with the definition of strict semantic rules, allows us to overcome this shortcoming (Mayr & Thalheim, 2021). The article states that in the natural and engineering sciences, the first step is to conceptualize the object (the problem to be solved) and then move on to modeling and studying behavior. The very fact that abstract elements such as class, attribute, relation, domain, state, etc. are defined

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confirm the importance of this initial stage in model design. In this reason, the conceptual model of a real system of process represents some image of it in an environment of terms, assumptions and hypotheses.

Conceptual modeling is applied in different fields and has different meanings related to the problem being solved. Nevertheless, in most cases it refers to the development of a model for the organization of a research process or the development of a prototype or system. An example in this direction that can be used as an illustration of the applicability of conceptual modeling is presented in (Fernandes et al., 2019). There, the concept of developing a system of information systems, representing dynamic associations of independent but interoperable information systems, is discussed. The idea is to set a common goal to achieve by sharing resources. To clarify the characteristics of such a complex system of systems, a conceptual model is proposed in the article, which, in addition to assisting the research and development of a description of the unified system, allows to make a correct classification of the various types.

As it was specified, for the creation of a conceptual model, mathematical means are applied, through which a formal description is created in a selected mathematical system. In the case of complex systems, a preliminary ***decomposition of the system*** into a finite number of subsystems is necessary, preserving the general functional algorithm. If necessary, subsystems can also be decomposed into elements convenient for formal description. Thus, relatively independent components of the general system with minimal information connections and interactions are identified. These components can easily be formalized independently.

Decomposition is a frequently used approach in system design because it allows the functions of individual sub-systems to be more easily realized. There are enough examples of active application of decomposition, some of which are presented below.

- Application in tree decomposition of a constraint system with adaptation of tree decomposition to work with parametric variables (Thibault, 2022).
- Development of a distributed framework based on the graph algorithm for calculating a control invariant set for non-linear cascade systems using the structure of interconnections in the process network (Decardi-Nelson & Liu, 2022).
- Use in the development of an algorithm for the assessment of the static state of energy systems with the expansion of the limits of resistance to deviations and cyber attacks (Ahmadi et al., 2021).

From a functional point of view, decomposition is the opposite of composition. In discrete mathematics, composition is an operation on representations and sets. Let us consider the set of functions over a set of elements M: $F_M = \{f \,/\, f: M \to M\}$.

The operation φ_o: $F_M \times F_M \to F_M$ is defined by the expression $f \circ g = h(x) = f(g(x))$, $h \in F_M$ and is called the composition of the functions f and g. The composition of functions is an associative operation, i.e., if $f,g,h \in F_M$, then:

$$((f \circ g) \circ h)(\mathrm{x}) = (f \circ g)(h(\mathrm{x})) = f(g(h(\mathrm{x}))) = f((g \circ h)(\mathrm{x})) = (f \circ (g \circ h))(\mathrm{x})$$

or for short:

$$f \circ g \circ h = (f \circ g) \circ h = f \circ (g \circ h) = f(g(h(\mathrm{x})))$$

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