Chapter 12
What to Assess for Mathematical Writing:
The Dimensionality of Written Expression for Mathematical Problem Solving and Reasoning

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ABSTRACT

This chapter investigates the internal structure of informative/explanatory mathematical writing (MW) as a latent variable. The study defines it as written communication in mathematics to explain and describe one's problem solving and reasoning by using mathematical language and representation. To assess the dimensions of MW, ten scales were developed based on literature and expert reviews to quantify students' MW performance. The dataset utilized for statistical analyses consisted of MW samples from 153 4th-grade students attending a public school in a Mid-Atlantic state. Through exploratory factor analysis (EFA) conducted on the three MW prompts, two factors emerged: mathematics problem solving and reasoning and writing. Subsequently, a confirmatory factor analysis (CFA) was conducted, resulting in a cross-factor loading of reasoning onto both factors. The role of representations in MW remains uncertain. This finding contributes to an understanding of the underlying dimensions, as well as offers a validated approach for assessing MW.

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INTRODUCTION

Written communication for mathematical thinking and reasoning is one of the key standards for mathematics education (National Council of Teachers of Mathematics, 2000; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). According to the National Council of Teachers of Mathematics (NCTM), students should learn to communicate their mathematical problem solving, justify mathematical conclusions, and evaluate mathematical arguments (NCTM, 2000). The Common Core State Standards for Mathematics (CCSSM; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) requires students to "construct viable arguments and critique the reasoning of others" (p. 6). In line with the curriculum standards, many state assessments in mathematics (e.g., Connecticut, Illinois, Massachusetts, Michigan, New York, Pennsylvania, Texas) include one or two open-ended items asking students to (a) explain their *mathematical problem solving* and (b) justify their *reasoning* in a logical and coherent sequence with written language (Hughes et al., 2019).

In response to the increasing demands to incorporate mathematical writing (MW) into the mathematics curriculum, numerous researchers have sought to clarify its definitions and dimensions. According to Baxter et al. (2005), MW is a writing activity "to explain and justify a (mathematical) problem solution" (p. 120). Similarly, Cross (2009) and Cohen et al. (2015) defined MW as writing in mathematics to explain how to *solve* a mathematical problem and *justify* reasoning and procedures. Kiuhara and her colleagues (2020) defined MW as a learning activity (Klein, 1999) to use *mathematical language* to justify the mathematical conclusions that students made. Lepak et al. (2018) placed substantial emphasis on the significance of *pictorial* and *symbolic representations* in written mathematical reasoning. These representations serve as a means for mathematical communication, visualizing mathematical relationships between salient quantities within the mathematical context of the given problem statement.

Similar to other forms of content writing, MW employs various types of writing to serve different purposes. Casa and colleagues (Casa et al., 2016; Colonesse, et al., 2018) described four genres of MW: (a) exploratory, (b) informative/explanatory, (c) argumentative, and (d) creative writing. The most frequent type of MW prompts in students' textbook and the state assessments is informative/explanatory writing (Casa et al., 2016; Colonesse et al., 2018; Hughes et al., 2019). Informative/explanatory writing in the context of problem solving describes the reasoning used to find a solution to a problem, highlighting the understanding of mathematical concepts and representations used. From these perspectives, the current study focuses on informative/explanatory writing in mathematics, specifically defining it as written communication in mathematics wherein individuals explain their own problem solving procedures and reasoning by using mathematical language and representation (Bossé & Faulconer, 2008; Hughes & Lee, 2020; Namkung et al., 2020).

Based on our definition of informative/explanatory MW, the purpose of this chapter is to explore the dimensionality of MW as a latent variable. Over the past two decades, there has been growing interest to explore *how to effectively teach* MW for students at various grade levels and verify their effectiveness. However, relatively less attention has been devoted to understanding *what to assess* in students' MW performance (Powell et al., 2017). To begin to address this question, this chapter focuses on (a) building and validating a hypothesized model to explain the internal structure of MW and (b) discussing what components should be assessed based on the quantitative results.