Chapter 4
Quantum Entanglement

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ABSTRACT

The aim of this chapter is to introduce the reader to various aspects of quantum entanglement. A detailed summary of the jargon of important mathematical notions, and concepts used in entanglement theory is provided. Various methods of entanglement generation are discussed followed by an introduction to the methods of detection and quantification of quantum entanglement in terms of various witnesses and measures. A brief account of some well-known applications of quantum entanglement is presented. Finally, a set of problems with solutions is provided, illustrating various important concepts discussed in the chapter.

INTRODUCTION

The notion of quantum entanglement first arose in a thought experiment put forward by Einstein, Podolsky and Rosen (Einstein, Podolsky, & Rosen, 1937), and essentially means that one cannot describe the joint quantum systems in terms of just local descriptions. The consequences of this seemingly simple idea are not so simple, as it defies our basic understanding of the knowledge about a system. A complete information about an entangled system does not mean a complete information about its constituent parts. A simple example is a pair of electrons in a singlet state, which can be written in Dirac notation (barring normalization) as $|\psi\rangle = |u_n d_n\rangle - |d_n u_n\rangle$, in which we know for sure that the angular momentum (called spin) of the total state is zero, but we have no knowledge of the individual angular momenta. The letters u_n and d_n stand for the spin being "up" and "down" along a particular direction n. As a consequence, measuring one part of an entangled system instantaneously affects our knowledge about the other. This tempts one to think about information transmission

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at faster than the speed of light– called as *spooky action at a distance* by Einstein, since he believed that for two particles to remain in contact over arbitrary great distance would need them to communicate at the speed greater than the speed of light, which is not allowed by the special theory of relativity. However, with our current understanding, correlations can exist without communication and entangled particles should be thought of as a single system. The experiments have proved beyond any doubt that entanglement is real and its presence over hundreds of kilometers has been demonstrated. With the recent developments in quantum computation and communication, *entanglement* has found itself at the heart of various application such as teleportation (Ekert, 1991) (Bennett C. H., et al., 1993), cryptography (Bennett & Brassard, 1984), dense coding (Bennett & Wiesner, 1992), and plays a key role in many body phenomena such as superconductivity (Shi, 2004), quantum phase transition (Osterloh, Amico, Falci, & Fazio, 2002) (Vidal, Latorre, Rico, & Kitaev, 2003) or fractional quantum hall effect (Kitaev & Preskill, 2006). There are various challenges for working with entangled states. An entangled system is very fragile, in the sense that the entangled particles become entangled also with their surroundings and this process is very quick, destroying the original entangled state one started in the first place. One of the challenges is to control the entangled systems in a way that allows the entangled particles to interact with themselves and prevents them from interacting with the environment, thereby preventing errors to crept in while carrying out various quantum computation tasks (Banerjee, 2019) (Naikoo, Dutta, & Banerjee, 2019) (Thapliyal, Pathak, & Banerjee, 2017).

The chapter is organized as follows: A brief overview of various mathematical notions and some important concepts, including the entanglement beyond bipartite scenario, is presented in the next section. This is followed by a brief account of various methods of entanglement generation. Next, the reader is introduced to methods of detection and quantification of entanglement, the notions like maximally entangled mixed states and the entanglement breaking channels. Some well known applications of quantum entanglement are discussed followed by various illustrative examples.

MATHEMATICS OF ENTANGLEMENT

We start this section by listing some basic properties of linear mappings in the context of Hilbert space $\mathscr{H}$.

Definition 1. *A mapping $\mathscr{A}:\mathscr{H} \rightarrow \mathscr{H}$, is linear if it preserves linear combinations:*

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